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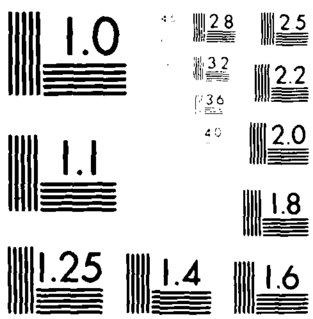

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STATISTICAL ANALYSIS OF STRENGTH
AND LIFE OF COMPOSITE MATERIALS

Pei Chi Chou, A.S.D. Wang, Robert Croman,
Harry Miller, and James Alper

Dyna East Corporation
Wynnewood, PA 19096

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1 Apr 77 - 30 Oct 79

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This technical report has been reviewed and is approved for publication.

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20. ABSTRACT

Static strength and fatigue life of unidirectional graphite/epoxy composites are studied in terms of statistical models of fatigue degradation and residual strength. A two-segment Weibull distribution for fatigue life, and a cumulative damage rule for fatigue are introduced. Experimental tests performed support the assumption that a specimen's rank in strength is equal to its rank in fatigue life. Fatigue weakens the matrix, and thus degrades the residual compression strength, and not the residual tension strength in early life. In some cases, residual tension strength initially increases. Proof tests can guarantee a minimum strength, and to a lesser extent, a minimum fatigue life.

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Foreword

This is the final technical report of a program sponsored by Materials Laboratory, Air Force Wright Aeronautical Laboratories, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio 45433, under contract F33615-77-C-5039 with Dyna East Corporation. The Air Force monitor was Mr. Marvin Knight, and the work was done under Project 2419 "Nonmetallic Structural Materials," Task 241903 "Composite Materials and Mechanics Technology." Dr. Pei Chi Chou was the principal investigator. He was assisted by Dr. Robert Croman and Mr. Harry Miller, in the theoretical analysis. The experimental phase of the research was carried out by Dr. A.S.D. Wang, with the assistance of Mr. James Alper.

The project was for a duration of 30 months. The work was performed during the period April 1, 1977 to October 31, 1979

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Statistical Analysis of Strength and Life of Composite Materials

I. Introduction

The objective of this program is to analytically simulate the static and fatigue failure of composites in terms of relevant statistical models. This model simulation is then confirmed by actual static and fatigue tests. The results presented here will help in achieving the long range objective of using composites with a higher degree of confidence and to remove the unnecessarily high safety factor because of the high scatter in strength and life properties.

Due to the large scatter, the properties of composite materials, such as static strength and fatigue life, must be treated as random variables. In conducting experiments, a statistically meaningful number of specimens must be used for each test condition. Statistical tools must be used in interpreting the test results. For a complete understanding of failure mechanisms, fracture mechanics and other deterministic failure models must be used. We need to know what is the failure mode (fiber breakage, debonding, matrix degradation, cracks in matrix, etc.) Deterministic models alone, however, are not sufficient because they cannot be verified by experiments, unless statistical methods are incorporated. Therefore, both the deterministic mechanics approach and the statistical approach are essential in the complete understanding of failure of composites. The emphasis of the present program is the statistical approach.

We shall make the general assumption that the failure modes under fatigue loading are essentially the same as those under static loadings of the same nature. In other words, fatigue failure under tension fatigue is similar to static failure in tension. This major assumption, if proved to be correct, will have major implications both in our basic understanding and practical application

of composite materials. If the failure modes of static and fatigue loadings are the same, then it follows that a particular specimen that is strong in static strength must also have long fatigue life. The words strong and long are both used in relation to other specimens of the same nominal specification. In more precise terms, we may say that among a population of specimens the ranking of a particular specimen in static strength must be the same as its rank in fatigue life. We have called this the strength-life equal-rank assumption. It is shown that this assumption is implied in many existing fatigue degradation models.

In support of the equal-ranking assumption, we have studied the residual strength after certain cycles of fatigue loadings. Both experimental tests and theoretical models are used. We have also made proof-test experiments to support the equal-rank assumption. A static proof load is first applied; the surviving specimens are then tested either in static strength or fatigue life. Our experimental results seem to support the equal-rank assumption.

In the theoretical modeling, we have proposed a sudden-death model, which should be used to distinguish degradation of residual strength. Three possible regimes of residual strength are identified. A specimen may have an increase in strength, weak degradation, or strong degradation. It is pointed out that some investigators mistake the weak degradation as an increase of strength.

It is shown that the increase in residual strength we observed, as well as similar cases observed by a few other investigators, is statistically meaningful and represents a true increase in strength. This is explained in terms of fracture mechanics.

Another approach in studying the failure mechanism under tension fatigue is to study the residual compression strength after tension fatigue. Our experimental results show that tension fatigue degrades the compression residual strength much more than it degrades the tension residual strength. This gives support to the assumption that tension fatigue weakens the matrix more than the fiber.

During the course of our study, we found out that the existing statistical tools are not sufficient for composite fatigue research. For instance, because of non-uniform definition of terminology, the equations and curves in most reports cannot be reproduced by others. The fatigue life distribution is of such a particular form that existing simple types, such as log normal and Weibull, cannot be fitted. For these reasons, we have devoted a certain amount of effort in developing statistical tools for the study of composite fatigue. We have assisted in organizing a Workshop on "Statistical Aspects of Testing of Composite Materials." In addition to introducing basic statistical concepts to composite engineers, we have tried to establish a general convention of data reduction and presentation. This will be helpful in communication between different investigators. Also, some recently developed methods were presented.

Another statistical tool we developed is on fitting distribution equations to fatigue data of composites. It is shown that the fatigue life of certain composites is best fitted by two segments of Weibull distribution. Maximum likelihood estimation equations are developed for fitting this two-segmented Weibull distribution.

Most of our results have been reported either in AFWAL/ML Technical Reports [1] [2], or in open literature [3] [8]. The details will not be repeated here. Some of the data, curves, and discussion have not been reported before; these will be included in this report.

In Section II, the results of our theoretical research are summarized. In Section III, our experimental work is outlined. Two of our recently presented papers are included in the Appendices of this report for easy reference.

II. Theoretical Analysis

Degradation and Sudden Death Models

Two models for the residual strength in fatigue tested specimens are studied, namely, the degradation model and the sudden-death model. The degradation model is based on an assumption used previously [9]-[11] which stipulates that the strength of each specimen decreases a little after each cycle of fatigue loading. When the residual strength drops to the value of the applied fatigue stress, fatigue failure occurs. On the other hand, the sudden-death model assumes that the strength of a specimen does not change after each cycle of fatigue loading. The effect of each cycle is impressed on the specimen in a form other than reducing its residual strength. For instance, it may change the matrix properties, which does not change the residual strength immediately. The fatigue failure is governed by some mechanism other than the residual strength. Only when the applied cycles are close to the fatigue life will the strength then drop drastically in a short number of cycles. For the sudden-death model, we have to impose the additional assumption that there is a unique relation between static strength and fatigue; the stronger ones last longer [4]. This unique relation is implied in the degradation model.

In comparing the residual strength with the static strength, it is proposed that the reduced population that includes only "top-percentage" of the static specimens should be used. The percentage that is excluded should equal the percentage of the fatigue specimens that failed before the residual strength test is taken. By comparing the "top-percent mean" of the static strength with the mean residual strength, we can see whether there is degradation or increase in strength. The top-percent mean can be calculated either from the distribution of the total population by taking proper conditional probability, or by taking the sample mean of the appropriate stronger samples.

The theoretical models were compared with six sets of experimental fatigue residual strength data of graphite/epoxy composites. These data were obtained from the works of four independent researchers. In general, the degradation model correctly predicts the mean of the residual strength of the six sets of test data studied. This is not a severe test for the model, because the decrease in the mean residual strength in these six cases is very small. The sudden-death model, which assumes no degradation for individual specimens, is also satisfactory in predicting the residual mean for most of these six cases.

The degradation model presented here is overly restrictive. Once the static and life distributions are given, it predicts a fixed residual strength distribution, which may not agree with experimental data. A more general degradation model with an additional parameter would be more versatile in matching different residual strength distributions of composite materials.

It is also found that for unidirectional composites, the decrease of residual strength is less than that for composites of general layup. In fact the residual strength may increase initially [3].

An Improved Degradation Model

Realizing that the degradation model of reference 3 is overly restrictive, an improved degradation equation which contains an additional parameter was introduced and presented in reference 4. This parameter can be adjusted to fit various residual strength data for a material of fixed static strength and life distributions.

Hahn and Kim [9] have introduced the assumption of a unique relation between the ranks in static strength and in fatigue life of a given specimen.

We shall call this the strength-life equal rank assumption. This is a very fundamental assumption because if this is not true, then the equation of degradation of residual strength cannot be deterministic, and must involve random variables. It must be pointed out that almost all fracture mechanics equations in fatigue are deterministic and they all imply this equal rank assumption.

As mentioned in the Introduction, the equal-rank assumption is a direct consequence of the similarity in failure mode between static and fatigue failure. We may mention here that this similarity exists for metals. For steel, it is now generally accepted that the fatigue limit is related directly to its tensile strength. The empirical relation is that fatigue limit strength is one-half the ultimate tensile strength. It is easy to hypothesize from this relation that for steel, the failure modes under fatigue and static tension are similar.

In reference 4 we first make the strength-life equal rank assumption. Based on this assumption, the constraints on the degradation equation are derived. Then a possible form of degradation equation is introduced. The residual strength distribution is then derived, and compared with existing experimental results. For the test data on graphite/epoxy studied, two sets show increase in residual strength, two show weak degradation, and two show strong degradation.

Another concept introduced here concerns two processes going on simultaneously during fatigue. One is the degradation of individual specimens, the other is the weeding out of weak specimens by fatigue failure. Depending on which of these two processes is more dominant, the mean of the residual strength can be smaller or larger than the mean of the static strength. The former is called strong degradation, the latter weak degradation.

<u>Two-Segment Weibull Distribution</u>

In studying the test data of fatigue life distribution of composites we have observed that in certain cases, the distribution is best represented by two distribution functions, one in the low life region, one in the high life region.

In reference 5, we have presented the maximum likelihood method of estimation of parameters of a two-segment distribution where each segment is a two-parameter Weibull distribution. It seems that the maximum likelihood estimation method has not been applied to the two-segment Weibull distribution with unknown parameters previously. We first derive the equations of the M.L.E. method, with progressive censoring capability. The general approach is similar to that used by Cohen [12], who has applied the M.L.E. to a single Weibull function, with progressive censoring. A few illustrative examples are given. Two of these examples are for fatigue life of composite materials.

Details of this work is given in Appendix A.

A Cumulative Damage Rule

Traditionally, engineers have used Miner's rule for cumulative damage in fatigue. Miner's rule is a simple phenomenological rule, which gives the life under consecutive loadings of different load levels, if the life at each individual level is known. It is well-known that Miner's rule is deterministic; it gives only mean fatigue life and does not account for the variability, or distribution, of life. It is shown in reference 6 that Miner's rule contains basic statistical inconsistency, and cannot be subject to experimental comparison for composite materials because of their large scatter in life.

In reference 6, a new cumulative damage rule that is applicable for fatigue life with large scatter is proposed for composite materials. It is based on the concept of percent-failure instead of the percent-life consumed assumption implied by Miner's rule. This rule is just as simple as Miner's rule, is statistically correct, and can be subject to experimental comparison. The proposed cumulative damage rule is applied to two sets of experimental data. Both sets of data agree well with the proposed rule.

The Existence of Endurance Limits

In studying fatigue of any material, it is always desirable to know its endurance limit, also known as fatigue limit. For composites, the existence of an endurance limit has not been established because of lack of sufficient test data. This is partly due to the difficulties involved in fatigue testing of composites in comparison with testing of metals. The composite specimens are usually coupons subjected to tension loading. This is not as simple as the standard rotating beam fatigue tests used for metals. In addition, due to heating during fatigue, the maximum frequency applied to composites is limited to around 10 to 20 Hz, whereas for metals, a frequency of more than 100 Hz is very common. Because of the low frequency, high cycle fatigue data for composites are very time consuming to generate.

By studying the limited amount of existing fatigue data, we would like to speculate that there is no clear-cut endurance limit for composites. A clear-cut distinct endurance limit exists for ferrous metals because they possess workhardening property during the crack nucleation stage. For composites, we do not detect any work hardening property, therefore, a distinct endurance limit is not likely to exist.

In order to have a better perspective of the situation, let us review briefly the fatigue mechanism of metals. Fatigue behavior of ferrous metals such as steel, is quite different from that of nonferrous metals. The former has a distinct endurance limit, the later does not. For steel, enough tests have now been run at 10^9 cycles to establish that if the specimen does not break by 10^7 cycles, it would not fail if the tests were continued. Ferrous metals can withstand an infinite number of stress cycles providing the stresses are all below the endurance limit. This endurance limit is a "true" limit in

the sense that if the applied stress is below this limit no failure will occur, regardless of the number of cycles applied. Also, the S-N curve in log-log coordinates has two straight line portions intersecting at a "knee."

On the other hand, for nonferrous metals such as aluminum and copper, there appears to be no true endurance limit. The S-N curve does not show two straight line portions, but a continuous curve with no "knee." For these metals, it is customary to take the strength at $5x10^8$ cycles as the endurance limit, even though it may not be established that the S-N curve has become completely flat at this point. In these cases, the endurance limit is an arbitrarily defined strength for engineering convenience. We are not sure that at a lower stress the specimen will endure infinite cycles.

The existence of distinct endurance limit for ferrous metals can be explained, at least qualitatively, from the fracture mechanics point of view. It is generally accepted that fatigue failure of metals involves three stages, namely, the crack nucleation, the crack propagation, and the final fracture stages. During the crack nucleation stage, shear deformation (slip) occurs in the atomic crystalline structure. This slip action may develop into slip-bands and microcracks. During this stage, the slip and microcrack follows the direction of maximum shear stress. During the second stage, the crack propagation stage, the microcrack grows in length after each cycle of loading. This crack propagates along a direction normal to the maximum tension stress. When the crack is of sufficient length, the remaining intact material cannot support the load, final fracture and separation then occur.

During the crack nucleation stage, if the applied stress is low, the reversed cyclic loading may cause work hardening, or precipitation hardening.

This hardening strengthens the local vulnerable spots sufficiently so that the slip action ceases. It is believed that ferrous metals do have this microscopic work hardening effect. When the applied stress is below the fatigue limit, the micro-work hardening strengthens the material, and any subsequent cyclic loading at the same stress will not nucleate any microcrack, and the material will not fail under fatigue.

For nonferrous metals, it does not possess this work hardening property, and thus under a very large number of cycles, crack nucleation and the resulting fatigue failure can still occur.

Coming back to composite materials, we feel that long time fatigue tests should be performed to establish the existence or non-existence of a true endurance limit. But from the practical point of view, perhaps a convenient number of cycles should be selected, and define the strength at that cycle as the endurance limit, just as in the nonferrous metal case. The cycle may be less than 5×10^8, because of practical limitation.

Increase of Fatigue Residual Strength

A few investigators have reported the increase of residual strength after fatigue loading for certain composite material laminates. Most of these observations cannot be used as clearcut evidence, because the experimental difficulties involved. For a given specimen, we can measure either its static strength, or its residual strength, but not both. By nature, the strength measurement is destructive, for one specimen only one strength can be measured. Therefore any measurement concerning residual strength change must resort to average values of a group of specimens. It is customary to compare the mean residual strength of a group of specimens to the mean static strength of a similar group. If the mean residual strength was higher than the mean static

strength, it was assumed all specimens had increased in strength during fatigue. We have shown [4] that this practice could lead to erroneous conclusions. It was proposed that an increase of strength had occurred only when the mean of residual strength was higher than the mean of the "top percent" of the static strength specimens. This top percent mean can be obtained by the "sudden-death" model. If the mean residual strength was less than the top percent static mean but greater than the static mean of the total population, then weak degradation of the specimens was said to have occurred. Strong degradation happened when the residual strength mean was less than the total static strength mean. It is felt that comparing the residual strength mean with the mean of the top percent of the static strength leads to fairer conclusions.

Let us summarize a few of the experimental works that involves an increase of residual strength. Reifsnider _et al_. [13] measured residual strength of boron/epoxy laminates with layup of $[0, \pm 45, 0]_s$, and with a central hole. They subjected each specimen to fatigue loading, and stopped the loading when the change in dynamic stiffness is 18%. Thus, their specimens are subjected to different cycles of loading, but the same stiffness change. After this cyclic loading, static residual strength was measured. Their results show that the specimen with higher applied cycles has higher residual strength. They concluded that fatigue loading increases the residual strength. We believe, however, their experimental results are less than conclusive, because due to the large scatter of strength among specimens, it is possible that the stronger specimen takes more cycle to reach 18% stiffness change; the higher residual strength may not be due to fatigue, but merely indicates that it is a stronger specimen to start with.

Kulkarni _et al_. [14] also observed an increase in residual strength after tension tension fatigue in notched boron/epoxy laminates. Their results for

notched (circular hole) coupons of $[0_2/\pm 45]_s$ layup with maximum fatigue stress equal to 80% of the static strength, and R = 0.1, show an increase of residual strength of 10% after 5×10^4 cycles. The increases are 15% after 5×10^5 cycles, and 16% after 1.5×10^6 cycles. They used only three specimens for each case, and did not take into account the specimen that failed during fatigue. Although their analytical predictions based on transverse crack propagation agree with the experimental results mentioned above, they realize that there is no statistically significant experimental data base to form a positive conclusion.

Waddoups et al. [15] observed an increase in residual strength in $[0/90]_s$ graphite-epoxy composite laminate with circular notch, subjected to fatigue loading. They only gave average strength data, and did not indicate the number of specimens used and the number of specimens failed during fatigue, before the number of cycle for residual strength measurement is reached. We are not sure about the statistical significance of their results.

A few investigators have offered fracture mechanics explanations for the increase of residual strength. Zweben [16] offered an explanation for notched composite specimens. He studied the 0° layer of a composite laminate subjected to an axial loading. The notch is perpendicular to axial direction and cuts n fibers. At the root of the notch exists an intact fiber. Because of the high inplane shear stresses at this root, the matrix between the notch and the first intact fiber is likely to fail. He found that the stress concentration was highly dependent on the length of the matrix failed region in the fiber direction. The larger the region of failure, the smaller the stress concentration. In a fatigue loading situation the fiber at the root of the notch might not fail but the length of the matrix failed region might continue to grow. Thus the stress concentration might decrease with applied fatigue cycles. In such a case it is not unexpected to find the residual strength to be higher than the static strength for some composites and layups.

Sendeckyj [17] also attributed the decrease in stress concentration as the reason for increase of residual strength. He discussed the fatigue failure of $[0°/45°/90°]_s$ graphite-epoxy laminates with central hole. Under fatigue loading, matrix cracks will occur in various laminas which eventually lead to a decrease in the stress concentration of the hole. As a result, the residual strength could increase.

We believe that the explanations offered by Zweben and Sendeckyj are plausible; from mechanics point of view it is possible to have an increase in residual strength. Furthermore, our analysis of our present test data [4] and the test data of Awerbuch and Hahn [18], indicate a true increase in fatigue residual strength in unidirectional graphite-epoxy composites.

III. Experimental Work

In support of the theoretical study, we have conducted static and fatigue tests of more than 600 composite material specimens. We shall give a general description of these tests here. Some of the details may be found in references 3-7.

Specimens

The material system used for this study is the AS-3501-05, with a nominal fiber content of 65% by volume. Test specimens are cut from panels supplied from the manufacturer directly. The dimensions of the tension test specimens are 0.084 cm thick (6-ply), 1.9 cm wide and 22.9 cm long with glass-epoxy end-tabs of 3.8 cm in length. Thus, the test gage length is 15.3 cm. The dimensions of the compression test specimens are 0.084 cm thick (6-ply), 1.9 cm wide and 3.2 cm long. Compression specimens used for residual strength testing are cut from tension specimens; one tension specimen cut into three compression specimens.

Static Test

The static tension and compression tests are conducted on a closed-loop Instron tester under room temperature ($\sim 21°C$) and ambient humidity (~ 0.6 R.H.) conditions. The loading rate selected is approximately 4000 lb/min (1800 kg/min). The selection of test specimens follows a random number schedule. Wedges are used to position the top and bottom of the compression specimen properly in its gripping device. Between the top and bottom wedges there is a space of 0.64 cm.

Fatigue Life Test

The fatigue tests are also conducted on the Instron tester under the same room temperature and ambient humidity condition. No effort is made to monitor any temperature change in the specimen during fatigue. The loading procedure is as follows: the specimen is first loaded statically with manual control to the mean stress level; it is then subjected to oscillatory loading with the minimum to maximum stress ratio of $R = 0.1$. The running cyclic frequency is 9.5 Hz.

Most tests are carried to fatigue failure; some are suspended for purposes of either residual strength measurement, or the reduction of testing time.

Summary of All Tests

All the static and fatigue tests performed are summarized in Table 1. The experiments are grouped into 9 tests, 6 static and 3 fatigue. Table 1 gives a brief indication of the nature of each test, and the number of specimens used. The specific reference, or table in this report where details may be found is also given. In Test 1, base-line strength data are generated, both for tension and compression loading. Tests 2 and 3 are strength tests after proof-load to 0.88 S_m and 0.95 S_m, respectively, where S_m is the mean static strength. Tests 4, 5, and 6 are residual strength measurements, after fatigue loaded to the specified cycles (suspended at cycles). In the fatigue test, Test 7 is the base-line data generation at four maximum stress levels (0.61 S_m, 0.71 S_m, 0.81 S_m and 0.85 S_m). In addition, the last two columns of Test 7 in Table 1 show the cumulative damage tests at low-high and high-low loadings. Tests 8 and 9 are fatigue tests after static proof-loading to 0.88 S_m and 0.95 S_m, respectively.

Some of the test data that have not been reported before are given in Table 2.

Proof-Tests

The main objective here is to investigate the effects of proof-load on both the (post-proof) static strength behavior and fatigue life behavior of unidirectional graphite-epoxy laminates. The specimens are first subjected to a static load. Those surviving specimens are then loaded either statically, or under fatigue until failure. From the present test results, it is found that proof-loading does not change the essential features in the static strength. Proof-loading removes the weaker specimens from the population, thus guaranteeing a minimum strength for the specimens that survive the proof-test. Moreover, proof-loading degradates only slightly the fatigue property of the specimens; hence the procedure can still guarantee a minimum life, with a high degree of confidence.

Table 1. Summary of Number of Specimens used in Static and Fatigue Tests

(number in bracket gives Ref. or Table (Tab) where data appears)

(Total specimens tested - 624)*

Static Tests - total 317 specimens

Tests	Static Strength		Residual Strength Following 0.71 S_m (1.034 GPa) Fatigue**	
	Tension	Compression	Tension	Compression***
1 Base-line data	24,[Ref 7]	28,[Tab 2-A]		
2 Proof-load to 0.88 S_m (1.29 GPa)	25,[Ref 7]			
3 Proof-load to 0.95 S_m (1.39 GPa)	25,[Ref 7]			
4 Suspended at 10,000 cycles			15,[Tab 2-B]	60,[Tab 2-D]
5 Suspended at 100,000 cycles			15,[Ref 3]	60 [Tab 2-E]
6 Suspended at 1,000,000 cycles			8,[Tab 2-C]	57,[Tab 2-F]

Fatigue Tests - total 299 specimens

Tests	Max. Stress = 0.61 S_m (.885 GPa)	Max. Stress = 0.71 S_m (1.034 GPa)	Max. Stress = 0.81 S_m (1.179 GPa)	Max. Stress = 0.85 S_m (1.243 GPa)	Max. Stress = 0.71 S_m then 0.85 S_m	Max. Stress = 0.85 S_m then 0.71 S_m
7 Base-line Data	2,[Tab 2-G]	130,[Ref 7]	25,[Ref 7]	25[Tab 2-H]	20,[Tab 2-I]	22[Tab 2-J]
8 Proof-load to 0.88 S_m (1.29 Gpa)		25,[Ref 7]	25,[Ref 7]			
9 Proof-load to 0.95 S_m (1.39 Gpa)			25,[Ref 7]			

Notes:

* Eight of the 624 specimens tested were not reported due to a machine malfunction or an end tab failure.

** All residual strength specimens have been first subjected to fatigue.

*** Three compression specimens were made from each tension specimen.

Table 2. Summary of Test Results

A. Compression Static Strength, MPa. 28 specimens failed

524	669	738	828	904
566	711	780	835	1014
566	724	780	849	1021
580	738	787	897	1049
600	738	800	904	
669	738	814	904	

B. Residual Tension Strength, MPa, at 1×10^4 cycles. Tension-tension fatigued with max. stress = 1034 MPa (71% S_m). R = 0.1, f = 9.5 Hz, 15 specimens failed.

1276	1490	1580
1401	1490	1580
1407	1497	1587
1414	1497	1677
1456	1532	1684

C. Residual Tension Strength, MPa, at 1×10^6 cycles. Tension-tension fatigued with max. stress = 1034 MPa (71% S_m). R = 0.1, f = 9.5 Hz, 8 specimens failed.

1270	1504
1339	1525
1408	1601
1421	1622

D. Residual Compression Strength, MPa, at 1×10^4 cycles. Tension-tension fatigued with max. stress = 1034 MPa (71% S_m). R = 0.1, f = 9.5 Hz, 60 specimens failed.

428	573	621	676	718	773
442	586	628	676	718	773
483	586	628	690	724	780
483	593	628	697	724	787
497	593	635	697	738	787
497	600	649	697	738	807
524	621	656	697	759	814
538	621	656	697	759	828
552	621	669	697	766	856
573	621	669	697	766	862

E. Residual Compression Strength, MPa., at $1x10^5$ cycles. Tension-tension fatigued with max. stress = 1034 Mpa (71% S_m). R = 0.1, f = 9.5 Hz, 60 specimens failed.

297	511	593	656	724	780
442	518	593	669	731	780
462	518	600	669	731	787
476	524	600	676	731	787
476	524	628	683	738	800
483	524	628	683	752	842
483	559	642	690	759	842
504	566	656	697	766	856
511	573	656	718	773	883
511	580	656	718	773	890

F. Residual Compression Strength, MPa, at $1x10^6$ cycles. Tension-tension fatigued with max. stress = 1034 MPa (71% S_m). R = 0.1, f = 9.5 Hz, 57 specimens failed.

345	476	587	628	683	773
407	476	587	642	690	787
407	483	594	649	697	794
414	483	594	656	697	856
428	497	600	663	704	876
428	497	600	663	738	883
442	504	607	676	738	1035
442	524	614	676	738	
455	552	614	676	759	
462	552	621	683	766	

G. Tension-Tension Fatigue Life, Cycles. Max. stress = 885 MPa (61% S_m), R = 0.1, f = 9.5 Hz, 2 specimens suspended.

2(1,000,000)

H. Tension-Tension Fatigue Life, Cycles. Max. stress = 1243 MPa (85% S_m) R = 0.1, f = 9.5 Hz, 25 specimens, 4 suspended, 21 failed.

1	372	5,156	34,395	142,640
42	718	8,981	36,437	4(300,000)
196	1,955	12,648	40,799	
246	3,598	14,220	45,193	
300	3,824	23,118	49,297	

I. Cumulative Damage Fatigue Life, Cycles. Max. stress = 1034 MPa (71% S_m) for the first 458,900 cycles then Max. stress = 1243 MPa (85% S_m), R = 0.1, f = 9.5 Hz, 20 specimens, 3 suspended, 17 failed.

24,300	459,804	508,592	534,165	942,388
176,582	471,489	514,293	535,026	3(1,000,000)
350,702	479,189	521,446	693,471	
458,934	481,438	524,392	705,871	

J. Cumulative Damage Fatigue Life, Cycles. Max. stress = 1243 MPa (85% S_m) for the first 3,570 cycles, then Max. stress = 1034 MPa (71% S_m), R = 0.1, f = 9.5 Hz, 22 specimens, 12 suspended, 10 failed.

1	24	524,916
1	227	807,728
5	250	12(1,000,000)
13	472	

Residual Compression Strength

Both residual tension strength and residual compression strength results were obtained by suspending specimens from 0.71 S_m fatigue. Residual strength is the strength of a specimen after it has been fatigued to a predetermined life. Specimens were suspended at 10,000 cycles, 100,000 cycles, and 1,000,000 cycles. Figure 1 is a plot of median residual strength versus fatigue life. The median residual strength is nondimensionalized with its respective static median strength, and the fatigue life is nondimensionalized with its characteristic life.

It can be seen from Figure 1 that the median residual tension strength has not changed much (in fact, increased at two of the three suspended lives), whereas the compressive residual strength decreased up to 20%. This supports our hypothesis that cyclic fatigue loading weakens the matrix more than the fiber.

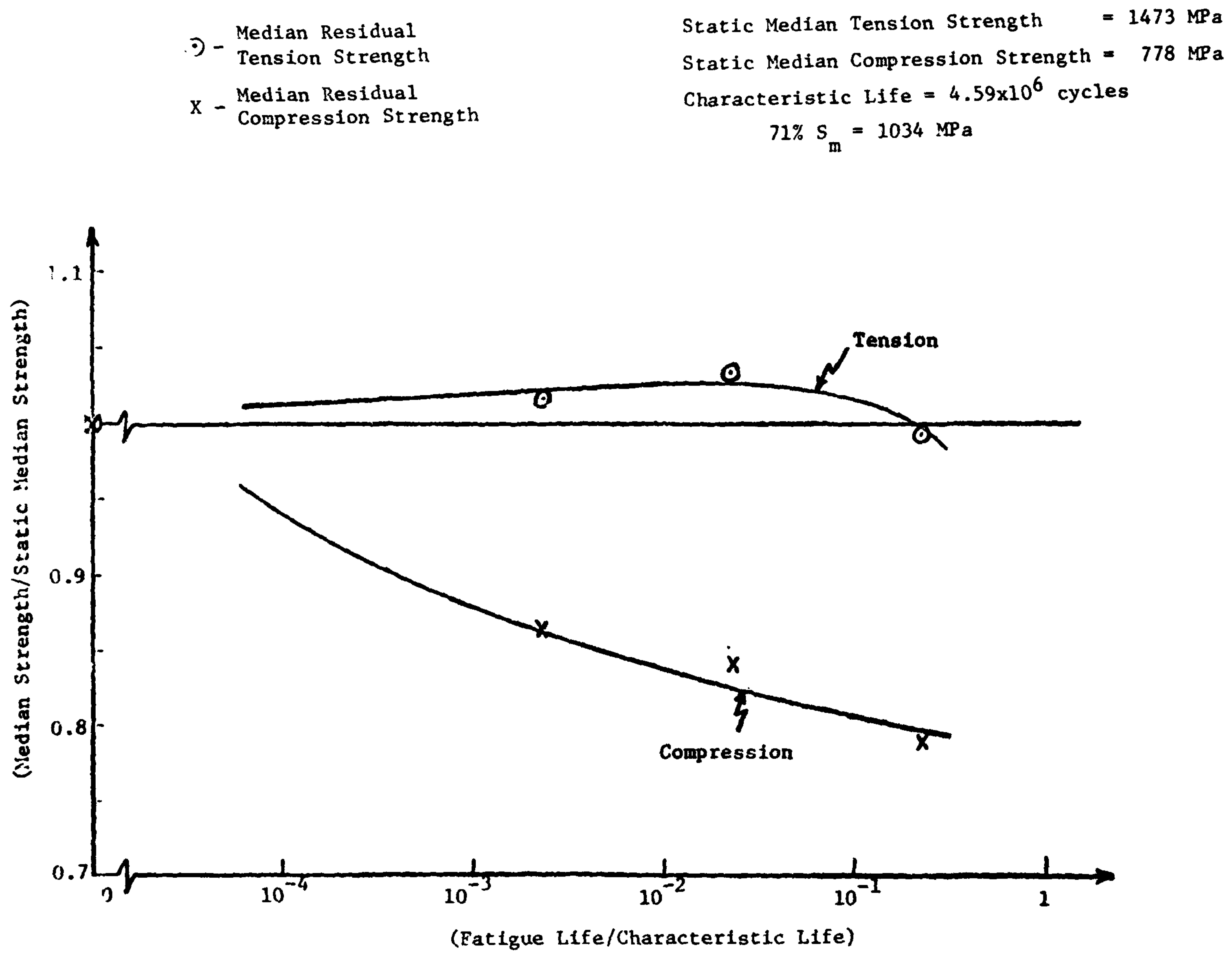

Figure 1. Median Residual Strength vs Tension Fatigue Life Maximum Cyclic Stress = 71% Static Mean Strength

Failure Modes

For specimens tested statically in tension, failure was always catastropic. In some cases the specimen was almost totally shattered leaving only a few longitudinal strands of material (see Figure 2), while in other cases there was fiber breakage forming two to three cracks normal to the longitudinal direction as shown in Fig. 3.

Figure 2. Shattered Unidirectional Specimen

Figure 3. Step-like Cracking in a Unidirectional Specimen

Failure in fatigue tested specimens was also catastropic. The specimens would show an initial fiber separation along the edges, but this does not lead to any failure. As the tests progress, some delamination and longitudinal cracks will appear. However, this does not indicate an impending failure. The specimens often survived long lives after the occurrence of a delamination or longitudinal crack. As with the static specimens, fatigue specimens either shattered or failed in a step like manner through the width of the specimen. Specimens failed in fatigue could not be distinguished from specimens failed statically.

For specimens tested in static compression, there were two modes of failure, end crushing and longitudinal matrix cracking (in the direction of the fibers). End crushing was the most common mode, and when end crushing occurred the specimens exhibited a sudden decrease in strength and the test was stopped. Specimens with high strength usually failed due to longitudinal cracking.

IV. Conclusions and Recommendations

In this section, we shall summarize the conclusions made in previous sections and make some recommendations.

1. Because of the large scatter of properties of composite materials, statistical tools must be used in interpreting test results. For a complete understanding of failure mechanism, both deterministic micromechanics approach and statistical approach are needed.
2. Residual strength of composite materials could increase after a small number of cycles of fatigue loading. This fact has been observed by a few investigators and is believed to be a statistically meaningful observation.
3. The curve of the sudden-death model proposed here should be used as the dividing line between the cases of increasing and decreasing residual strength.
4. The strength-life equal rank assumption seems applicable to composites under static tension and fatigue tension loadings. This implies that the failure modes for static and fatigue loadings are the same.
5. A degradation model is proposed. It satisfies the equal-rank condition and other theoretical constraints, and has a free parameter to adjust to various cases.
6. Our experimental results indicate that residual tension strength after a tension fatigue loading of less than the characteristic life of the specimen decreases very little, if any. Sometimes it may increase. However, the corresponding compression residual strength decreases appreciably from the static compression strength. Since compression strength is more dependent on matrix regidity, we conclude that during the early life period, fatigue loading degradates primarily the matrix.

7. Static proof-loading can guarantee a minimum static strength. To a lesser degree, it can also guarantee a minimum fatigue life.

8. Because composites do not possess work hardening properties, we speculate that a distinct endurance limit does not exist. We recommend that long time fatigue tests be performed to ascertain the existence or nonexistence of an endurance limit. For practical purposes, a specific fatigue cycle should be selected as the "apparent" endurance limit cycle.

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APPENDICES

Appendix A

Maximum Likelihood Estimation of a Two-Segment Weibull Distribution for Fatigue Life

This is the manuscript of a paper presented at the ASTM Symposium on Statistical Analyses of Fatigue Data, on October 30, 1979, in Pittsburgh, Pa.

Maximum Likelihood Estimation of a Two-Segment Weibull Distribution for Fatigue Life

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ABSTRACT: A two-segment distribution is proposed for the representation of fatigue life of modern high-performance composite materials. Each segment is a two-parameter Weibull distribution. Equations of the maximum likelihood method in estimating parameters are derived, and an iterative solution scheme is presented. Several example problems are included.

KEYWORDS: maximum likelihood estimation, Weibull distribution, censored samples, composite materials, fatigue, strength.

Nomenclature

k	subscript which denotes a segment ($k = 1,2$)
$F(x)$	cumulative distribution function
$f(x)$	probability density function
γ_k	Weibull shape parameter of the k^{th} segment
β_k	Weibull scale parameter of the k^{th} segment
θ_k	alternate form of the scale parameter ($\theta_k = \beta_k^{\gamma_k}$)
δ	value of x separating the two segments (i.e., intersection point)
N	total sample size; $N = n_1 + n_2 + n_3 + n_4$
n_1	number of failed sample points with value $\leq \delta$
n_2	number of failed sample points with value $> \delta$
n_3	number of suspended sample points with value $\leq \delta$
n_4	number of suspended sample points with value $> \delta$
x_i	value of the i^{th} failed specimen ordered such that $x_i \leq x_{i+1}$
y_i	value of the i^{th} suspended specimen ordered such that $y_i \leq y_{i+1}$

Introduction

Due to its light weight and high strength, modern high-performance composite materials, such as graphite fiber embedded in epoxy matrix, have been used as structural members in military and commercial aircraft. They are also being used in sporting goods (skis, tennis rackets and golf clubs) and are being considered as the structural material for the automobile in the 1980's. At the present, the composites have one disadvantage; that is, its strength and fatigue life have larger scatter than those of metals. Extensive testing is currently being carried out to characterize these materials for the purposes of better understanding their behavior, and for design applications.

In studying the test data of fatigue life distribution of composites we have observed that in certain cases, the distribution is best represented by two distribution functions, one in the low life region, one in the high life region. In this paper, we shall present the maximum likelihood method of estimation of parameters of a two-segment distribution where each segment is a two-parameter Weibull distribution.

Two-segmented distribution first appeared in Weibull's paper [1]. In introducing the distribution now bearing his name, Weibull considered two types of this distribution, a simple type, and a complex type. His simple type is a standard three-parameter Weibull; the complex type is the sum of two sub-populations. The distribution function of the complex type appears as two straight line segments in the Weibull coordinate. He showed a few examples, including one on the length of cyrtoideae (a kind of sea shell), and one on the fatigue life of steel; the latter one will also be used as an example for our present approach. He used three-parameter Weibull for each of his sub-population, and used trial-and-error method and curve fitting by eye in estimating parameters.

In 1959, Kao [2] also discussed the two-segmented Weibull distribution in connection with failure of electronic tubes. He proposes that the failure can be classified into two types; one is sudden or catastropic (infant mortality), the other is wear-out or delayed failure. The distribution function of life of the tube is the sum of two distributions, each of a sub-population. He called this a "mixed distribution," which is similar to the "multi-risk" model discussed by Herman and Patell [3]. Kao further demonstrated that when the characteristic life of the wear-out distribution is large, the mixed distribution can be approximated by a "composite distribution" which is essentially that of the two-segmented Weibull discussed in this paper. He used a 2-parameter Weibull for each segment, placed some restrictions on the values of the two shape parameters, and used graphical curve fitting in estimating the parameters.

Srivastava [4] studied the problem of life distribution of a specimen subjected to two alternating stress levels. He assumed that the Weibull shape parameter for these two stress levels are the same, but the scale parameters are different. The combined distribution for many alternating periods at each of the stress levels is derived.

In reliability engineering, the concept of multi-segment distribution is also being used. One example is the piecewise-linear failure rate (hazard function) model, which is one version of the well-known "bathtub curve," [5]. Mann, Schafer, and Singpurwalla [6] also discuss a 'two component composite distribution' which is similar to that described by Kao [2].

It seems that the maximum likelihood estimation method has not been applied to the two-segment Weibull distribution with unknown parameters. In this paper, we shall first derive the equations of the M.L.E. method, with progressive censoring capability. The general approach is similar to that used by Cohen [7], who has applied the M.L.E. to a single Weibull function, with progressive censoring. A few illustrative examples are given. Two of these examples are for fatigue life of composite materials.

Two-Segment Weibull Distribution

Let us consider the two-segment Weibull distribution with a probability density function defined as

$$f(x) = \frac{\gamma_k}{\theta_k} x^{\gamma_k - 1} \exp\left[-\frac{x^{\gamma_k}}{\theta_k}\right] \tag{1}$$

and the corresponding cumulative distribution function

$$F(x) = 1 - \exp\left[-\frac{x^{\gamma_k}}{\theta_k}\right] \tag{2}$$

where

$$k = 1 \quad \text{for} \quad x \leq \delta$$

and

$$k = 2 \quad \text{for} \quad x > \delta.$$

Each of the two segments is a two-parameter Weibull distribution. In general, at $x = \delta$, $f(x)$ is discontinuous but $F(x)$ is continuous. When the domain of x extends from 0 to infinity, we have the condition,

$$\int_0^\infty f(x)dx = 1 \tag{3}$$

which is equivalent to $F(\delta)_{k=1} = F(\delta)_{k=2}$.

Combining Eqs. (2) and (3), we obtain

$$\theta_1 = \theta_2 \; \delta^{\gamma_1 - \gamma_2} \tag{4}$$

This equation reduces the number of independent parameters from four to three. We shall consider γ_1, γ_2, and θ_2 as our independent parameters. In this study, δ will be treated as a preselected constant. The best suited value of δ will be determined by comparing the K-S statistics.

With a given value of δ, we shall use the maximum likelihood method in estimating the values of the parameters γ_k and θ_k for a random sample of N specimens containing (n_1+n_2) failed specimens, and (n_3+n_4) suspended, or censored, specimens. The censoring can be progressive, i.e., any number of specimens can be censored at any time.

The likelihood function for this distribution may be written as:

$$L = \text{const} \prod_{i=1}^{n_1} \frac{\gamma_1}{\theta_1} x_i^{(\gamma_1-1)} \exp\left[-\frac{x_i}{\theta_1}^{\gamma_1}\right]$$

$$\cdot \prod_{i=n_1+1}^{n_1+n_2} \frac{\gamma_2}{\theta_2} x_i^{(\gamma_2-1)} \exp\left[-\frac{x_i}{\theta_2}^{\gamma_2}\right]$$

$$\cdot \prod_{i=1}^{n_3} \exp\left[-\frac{y_i}{\theta_i}^{\gamma_1}\right] \cdot \prod_{i=n_3+1}^{n_3+n_4} \exp\left[-\frac{y_i}{\theta_2}^{\gamma_2}\right] \qquad (5)$$

On taking logarithms of Eq. (5), differentiating with respect to γ_1, γ_2 and θ_2, and keeping in mind that θ_1 is related to the other parameters by Eq. (4), we obtain the estimating equations

$$\frac{1}{\theta_1}\left[\sum_{i=1}^{n_1} x_i^{\gamma_1} + \sum_{i=1}^{n_3} y_i^{\gamma_1}\right] - n_1 = n_2 - \frac{1}{\theta_2}\left[\sum_{i=n_1+1}^{n_1+n_2} x_i^{\gamma_2} + \sum_{i=n_3+1}^{n_3+n_4} y_i^{\gamma_2}\right] \qquad (6)$$

$$0 = \frac{n_2}{\gamma_2} + \sum_{i=n_1+1}^{n_1+n_2} \ln x_i - \frac{1}{\theta_2}\left[\sum_{i=n_1+1}^{n_1+n_2} x_i^{\gamma_2} \ln x_i + \sum_{i=n_3+1}^{n_3+n_4} y_i^{\gamma_2} \ln y_i\right]$$

$$+ \frac{\ln\delta}{\theta_2}\left[\sum_{i=n_1+1}^{n_1+n_2} x_i^{\gamma_2} + \sum_{i=n_3+1}^{n_3+n_4} y_i^{\gamma_2}\right] - n_2 \ln\delta \qquad (7)$$

$$0 = \frac{n_1}{\gamma_1} + \sum_{i=1}^{n_1} \ln x_i + \frac{1}{\theta_1}\left[\sum_{i=1}^{n_1} x_i^{\gamma_1} \ln x_i + \sum_{i=1}^{n_3} y_i^{\gamma_1} \ln y_i\right]$$

$$+ \frac{\ln \delta}{\theta_1}\left[\sum_{i=1}^{n_1} x_i^{\gamma_1} + \sum_{i=1}^{n_3} y_i^{\gamma_1}\right] - n_1 \ln \delta \qquad (8)$$

Equations (4), (6), (7), and (8) form a system of four equations in the four unknown parameters γ_1, γ_2, θ_1, and θ_2. The solution for the parameters is obtained by an iterative scheme which involves a first estimation of the values of γ_1 and γ_2. These values are then substituted into Equations (4) and (6), and values of θ_1 and θ_2 are solved. These values are then used in Equations (7) and (8) to obtain new estimates of γ_1 and γ_2. This process is repeated until the values of all the parameters have converged.

The solution scheme presented above has been programmed for use on an IBM 370 computer. The convergence criteria used in this program compares the value of each parameter to its respective value in the previous iteration. If

$$\gamma_k - \gamma_{k(\text{previous})} < 0.0001 \qquad (9)$$

and if

$$\frac{[\theta_k - \theta_{k(\text{previous})}]}{\theta_k} < 0.0001 \qquad (10)$$

the values of γ_k and θ_k are considered satisfactory and the iteration process is stopped. For all data sets we have studied, convergence has always occurred within 40 iterations, even when the initial estimates were an order of magnitude higher than their final value.

Illustrative Examples

We shall present four examples. The first one is the fatigue life of Bofors ST-37 steel, which was originally studied by Weibull. The second one involves a set of data points taken from a known two-segment distribution. The last two examples involve the fatigue life of graphite-epoxy composite materials.

Bofors Steel

The data for this example is taken from Weibull's paper reference 1. Fatigue life data of 235 specimens of Bofors ST-37 steel under rotating beam tests were recorded. The life of individual specimens were not given; only the number of specimens failed within certain life intervals were tabulated. These are reproduced in Table A-1. In applying our M.L.E. equations, we have assumed that all specimens with life within a given interval have life at the upper limit of the interval. The results are shown in Fig. A-1. The data points are shown by vertical lines bounded by circles, the location of which is calculated according to the median rank formula [8].

Weibull's original fitted curve is also shown in Figure 1. He used two three-parameter Weibull distributions and fitted the curve to the data points visually.

In our solution, the value of the partition life, δ, is determined by comparing the K-S statistics of a few values of δ.

Figure A-1 is plotted in the "Weibull Coordinates"' two-parameter Weibull functions appear as straight lines, while three-parameter Weibull distributions do not.

Idealized Data Set

In the second example, we shall start with a hypothetical two-segment Weibull with known values of the parameters, select a few points from it, and then apply the M.L.E. to determine the parameters corresponding to these

points. These are then compared with the original distribution. The hypothetical distribution selected has the following values:

$$\gamma_1 = 2.0, \qquad \gamma_2 = 0.5$$

$$\theta_1 = 2.3 \times 10^5, \qquad \theta_2 = 4.0 \times 10^5$$

$$\delta = 1.913 \times 10^5,$$

Twenty points were selected from this distribution with equal ΔF between points. These points, together with the curve of M.L.E. of the distribution are shown in Fig. A-2. The agreement is satisfactory. It can be shown that as the number of points increases, the estimated values of the parameters approach the original value.

Fatigue Life of Composite Material, Complete Samples

In reference 9, Ryder and Walker tested graphite-epoxy composite laminates, typical to those used for aircraft structures. We shall study his data for fatigue life under tension-tension fatigue, of the Laminate II composites. Details of the specimen layup, testing condition, and fatigue life, are given in Table A-2. Twenty failed data points are available, which represents a complete sample without censoring.

The results are shown in Fig. A-3. The solid curve is the estimated two-segment Weibull, and the dotted line is a M.L.E. of a single function Weibull. The two-segment Weibull shows a good fit to the data.

Fatigue Life of Composite Material, Censored Samples

In reference 10, Wang, Chou, and Alper have studied the fatigue life of unidirectional graphite-epoxy composites. They used 25 specimens, 20 fatigued to failure, five suspended (censored) at 10^6 cycles. Their data are reproduced in Table A-3. The estimated distribution is shown in Fig. A-4. Again, the fit is satisfactory.

Table A-1 - Fatigue of Bofors Steel [1]
[Rotating Beam Test at $\pm$ 32 kg/mm^2]

Life, Cycles	Number of Specimens	Life, Cycles	Number of Specimens
12,500 - 17,500	5	47,501 - 52,500	6
17,501 - 22,500	43	52,501 - 57,500	4
22,501 - 27,500	78	57,501 - 62,500	3
27,501 - 32,500	44	62,501 - 67,500	2
32,501 - 37,500	23	67,501 - 72,500	1
37,501 - 42,500	14	72,501 - 77,500	1
42,501 - 47,500	8	77,501 - 82,500	1
		82,501 - 87,500	1
		87,501 - 92,500	1

Table A-2 - Ryder-Walker Tension-Tension Fatigue Tests [9]

Fatigue Life, Cycles: [max stress = 50 ksi, F = 10 Hz
Gr/E $(0/+45/90/-45_2/90/+45/0)_s$]

11,491	51,848	64,070	81,571
17,578	54,187	69,711	87,373
40,270	58,530	70,049	116,667
41,200	59,320	70,497	367,644
44,830	60,912	71,400	513,600

Table A-3 - Wang-Chou-Alper Tension-Tension Fatigue Tests [10]

Fatigue Life, Cycles: (6 ply Gr/E unidirectional)
(max stress = 171 ksi, F = 9.5 Hz)

30	288	5,984	15,754	1,000,000*
69	380	8,609	18,995	1,000,000*
90	1,570	11,362	22,515	1,000,000*
260	3,269	12,119	97,009	1,000,000*
286	5,653	15,529	149,356	1,000,000*

* suspended (censored)

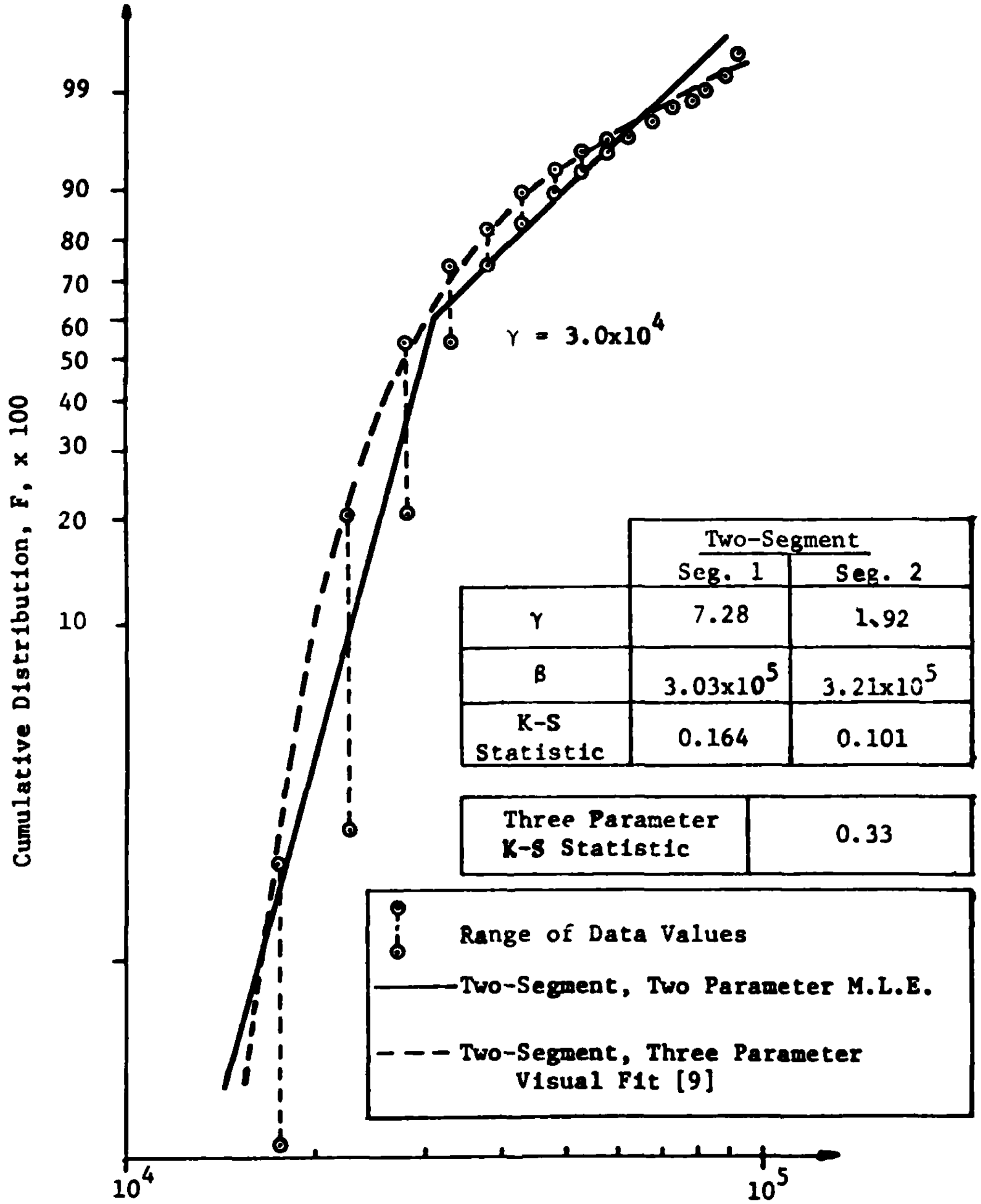


Figure A-1 Comparison Between the Experimental Data and the Two-Segment Distribution for Fatigue Life of ST-37 Steel. Reference 1.

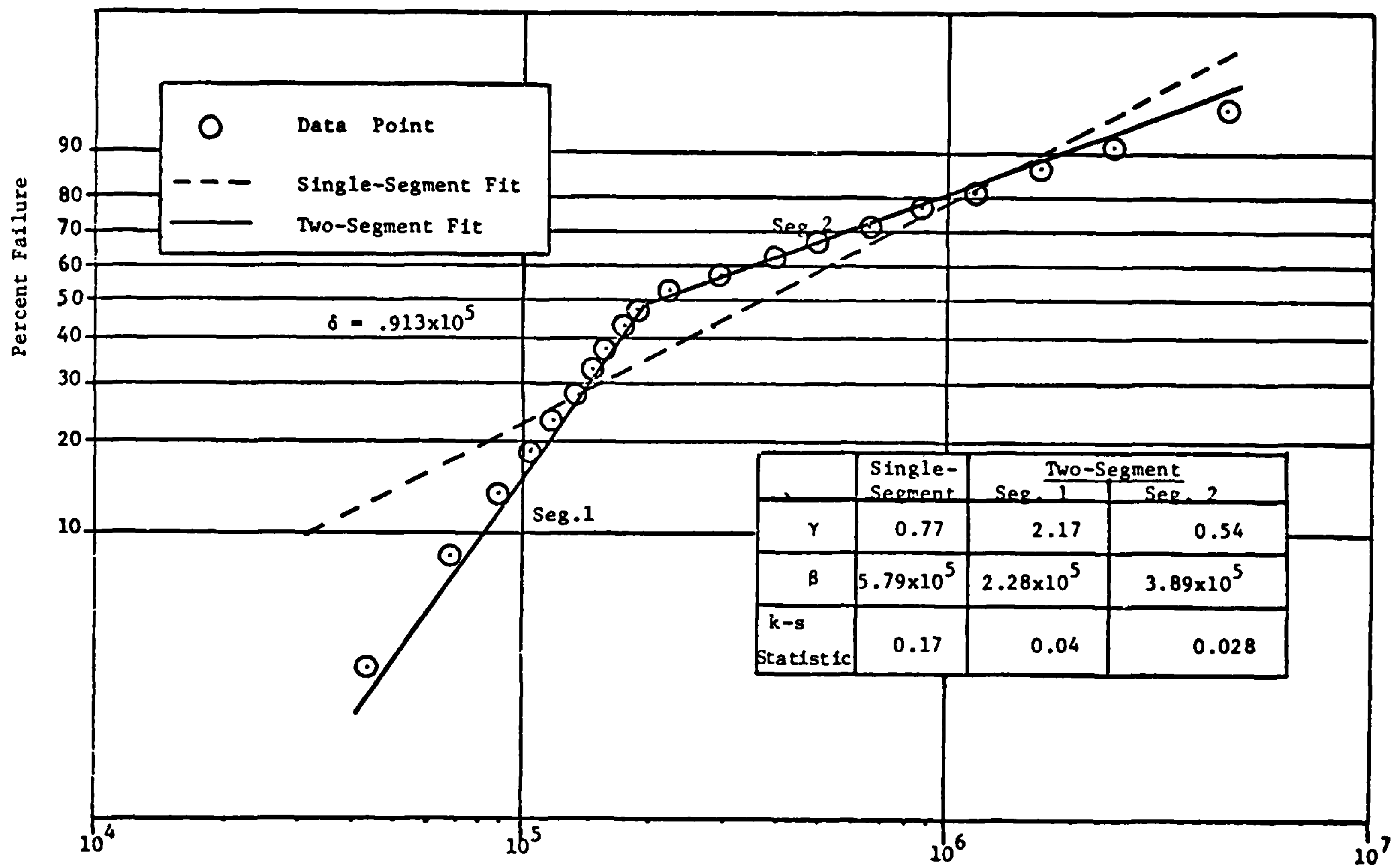


Figure A-2 Comparison between Two-Segment and the Simple Weibull Distributions for Simulated Data

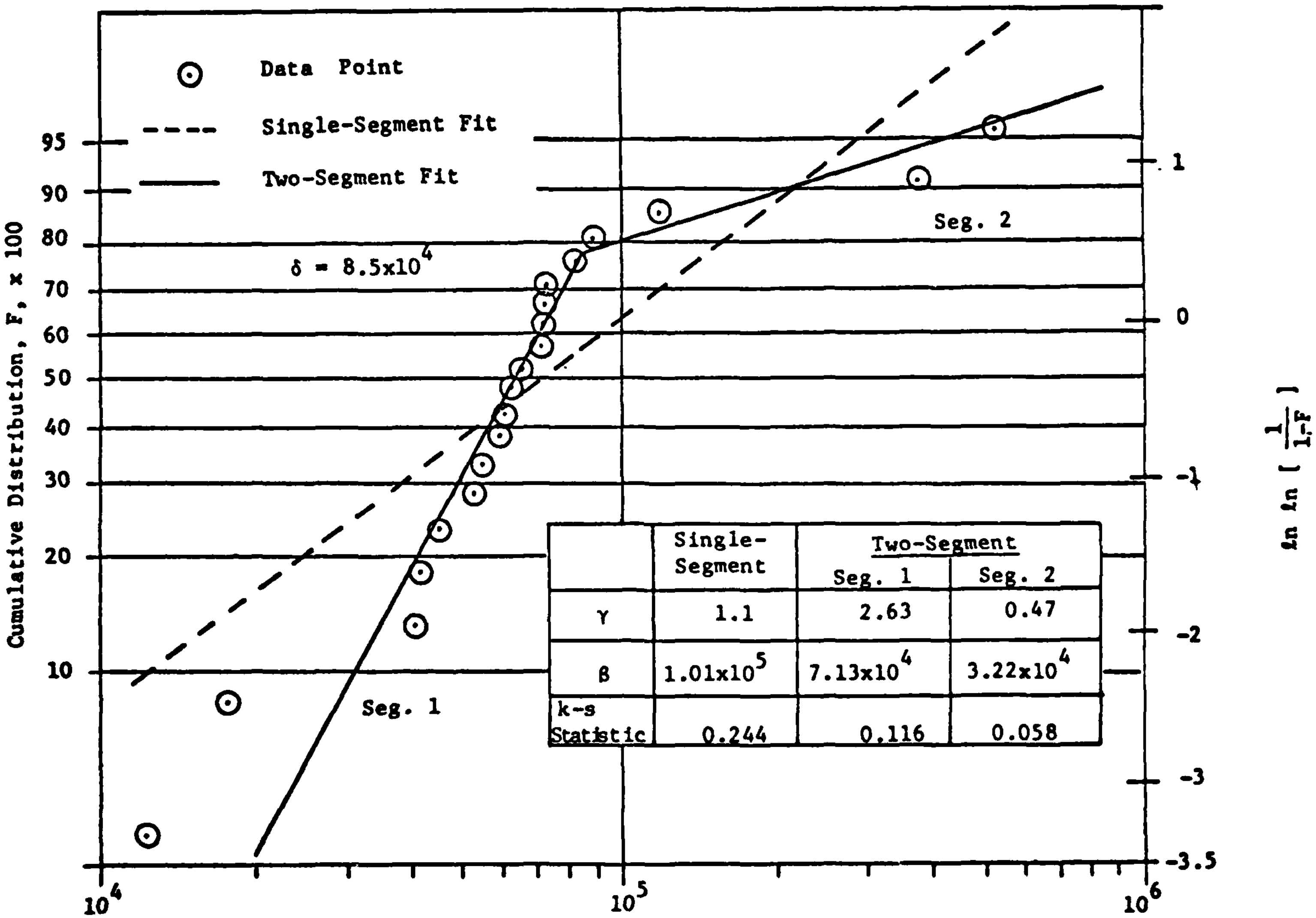
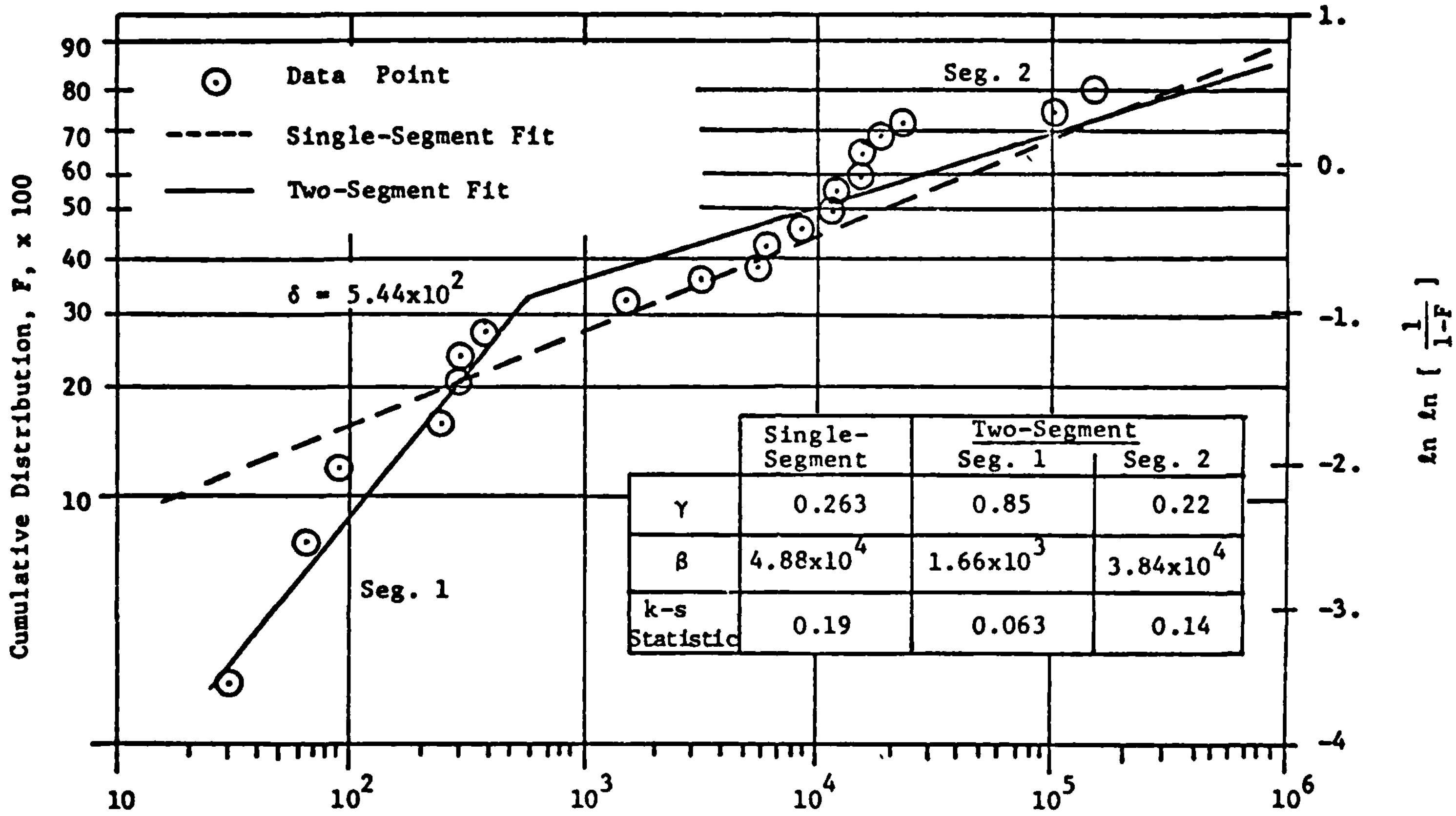


Figure A-3 Comparison Between the Two-Segment and the Single-Segment Weibull Distributions for Fatigue Life of Graphite/Epoxy Laminates. [9]



	Single-Segment	Two-Segment Seg. 1	Two-Segment Seg. 2
γ	0.263	0.85	0.22
β	4.88×10^4	1.66×10^3	3.84×10^4
k-s Statistic	0.19	0.063	0.14

Figure A-4 Comparison Between the Two-Segment and the Single-Segment Weibull Distributions for Fatigue of Unidirectional Composites. [10]

Concluding Remarks

We have shown that the fatigue life of certain materials can be best represented by a two-segment distribution, each a two-parameter Weibull. The maximum likelihood method is applied for parameter estimation with satisfactory results. We have not studied the failure mechanism or the cause of the failure. It is very likely that two separate failure mechanisms are present. The identification of the fatigue life distribution with two segments of distribution will facilitate the search for the failure mechanisms.

The three parameter Weibull distribution is often used to represent data that do not agree with a single two-parameter Weibull. With the present method of conveniently fitting a two-segment, two-parameter Weibull, it seems that there is no need in using the three-parameter Weibull. If the variable involved should have a domain from zero to infinity, like fatigue life, there is no physical reason to impose a finite minimum value, as the location parameter does in the three-parameter Weibull. Also, the two-parameter Weibull has the convenience that its shape parameter gives an indication of the degree of scatter in terms of the central value, just like the coefficient of variation. For the three-parameter Weibull, the shape parameter gives the degree of scatter in terms of the central value minus the location parameter, which is more difficult in making comparisons. For instance, in terms of two-parameter Weibull, the population which has the larger shape parameter has smaller scatter. This type of statement cannot be made for the shape parameter of the three-parameter Weibull. Further discussion on this point will be made in a separate paper.

Acknowledgment

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Appendix B

A Cumulative Damage Rule for Fatigue of Composite Materials

The contents of this Appendix have been presented in a paper of the same title at Winter Annual Meeting of the American Society of Mechanical Engineers, in New York City, on December 4, 1979. It is available in a bound volume entitled "Modern Developments in Composite Materials and Structures," edited by J. R. Vinson, ASME, New York, N.Y.

In this appendix, experimental results in addition to those reported in the ASME paper have been included.

A CUMULATIVE DAMAGE RULE FOR FATIGUE OF COMPOSITE MATERIALS

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ABSTRACT

A phenomenological cumulative damage rule for fatigue life is proposed. It is most suitable for composite materials because of the large scatter in fatigue life data. The rule is based on the percent-failure of a group of specimens, instead of the popular Miner's rule, which is based on the percent-life consumed. It is shown that Miner's rule contains statistical ambiguity and a precise comparison with experimental results cannot be made. The proposed percent-failure rule is compared with two sets of experimental results with good agreement.

NOMENCLATURE

E( )	-	expected value
F	-	cumulative distribution function
n	-	fatigue cycles
N	-	fatigue life
P	-	probability
S	-	fatigue stress level
x	-	fatigue cycles
X	-	random variable for fatigue life
y	=	$x - x_2$ = additional life at S_2 stress level.
α	-	Weibull shape parameter
β	-	Weibull scale parameter (characteristic life)
$\gamma^*(\ )$	-	incomplete gamma function
$\Gamma(\ )$	-	gamma function
μ	-	mean

Subscripts

1	-	at the first stress level; at S_1
2	-	at the second stress level; at S_2
ℓ	-	at the low stress level
h	-	at the high stress level

Introduction

It is well-known that modern high performance composite materials, such as graphite/epoxy laminates, have great variability and scatter in their properties, especially in fatigue life. For a group of nominally identical specimens under identical loading, the fatigue life can vary by a factor of 1000 between the most and least durable ones. Typical values of Weibull shape parameter for composites are around one or two; for unidirectional composites the shape parameter is even lower, and can be as low as 0.4. In comparison, metals have shape parameter typically above a value of four.

Due to this larger scatter and low value of shape parameter, it is essential that proper care must be taken in characterizing material properties. For instance, in specifying fatigue life at a given load level, just the mean life is not sufficient; instead, the life distribution must be given. The Miner rule for cumulative damage, which has been used for fatigue of metals, should be used with extreme caution when applied to composites.

Miner's rule is a simple phenomenological rule, which gives the life under consecutive loadings of different load levels, if the life at each individual level is known. It is well-known that Miner's rule is deterministic; it gives only mean fatigue life and does not account for the variability, or distribution, of life. It will be shown in this paper that Miner's rule contains basic statistical inconsistancy, and cannot be subject to experimental comparison for composite materials because of their large scatter in life.

Ever since its first proposal in 1945 [1], Miner's rule has been used widely for metal fatigue. People used it because of its simplicity. Its prediction of fatigue life is not always satisfactory in comparison with experimental results. Many modified versions of Miner's rule have been introduced; Leve [2] gave a detailed review of some of these. Basically, Miner's rule, together with most of the modifications, is based on the assumption that fatigue damage can be measured by the cycle ratio n_1/N_1, where n_1 is the number of cycles applied at stress level S_1, and N_1 is the fatigue life at S_1. This cycle ratio may also be considered as the "percent life" consumed. The original rule stipulates that when the sum of the cycle ratio reaches unity the specimen fails. The modified versions assume the damage is not linearly proportional to the cycle ratio, but a more complicated function of the cycle ratio.

Recently, cumulative damage in fatigue has been studied from a statistical, or probabilistic, point of view. Birnbaum and Saunders [3] studied the Miner's rule with a combined probabilistic and mechanics approach. They assumed that the fatigue failure is due to the growth of a dominant crack. The crack extension for each oscillation of loading is treated as a random variable. They further assumed that the crack extension (a) is independent of the current crack length, (b) has a distribution with an increasing failure rate (hazard rate), and (c) is statistically independent of the final failure crack length. Under these assumptions and by utilizing some results from renewal theory, they demonstrated that Miner's rule is applicable in terms of expected value (mean) of life. This is a very interesting approach in combining both fracture mechanics (crack propagation) and the statistical concept of random variables in studying fatigue damage. However, since it is based on the dominant crack propagation, its application to composite materials is of questionable validity; the failure mode in composites is much more complicated. Further, they did not give a comparison with any experimental data.

Bogdanoff [4] proposed a cumulative damage model which is basically a phenomenological model. He considered a certain time interval, called duty cycle (DC), as the basic unit for damage. The physical mechanism of damage accumulation of the material in this duty cycle is not considered. He then made statistical analysis by considering damage at the end of a duty cycle as a random variable, and accumulation of damage was treated by the Markoff Process. This is an interesting mathematical model of cumulative damage, but since it involves many assumptions and undetermined parameters, its practical usefulness at the present is limited. In comparing with experimental results, it can only be used to fit a life distribution function to test data; which can be done easily by other standard methods (such as fitting two or three parameter Weibull distributions by maximum likelihood method).

Yang and Jones [5] studied the effect of cumulative damage in fatigue life by assuming a specific equation for the residual strength degradation. In this equation, the stress level of the fatigue is a variable. By changing values of this variable, the cumulative damage at different stress levels can be accounted for. Their approach is essentially also a "percent life" damage rule, like the Miner rule. The results are limited to the cases where the particular degradation equation is applicable.

In this paper, a new cumulative damage rule that is applicable for fatigue life with large scatter is proposed for composite materials. It is based on the concept of percent-failure. This rule is just as simple as Miner's rule, is statistically correct, and can be subject to experimental comparison.

PHENOMENOLOGICAL CUMULATIVE DAMAGE RULE

We shall consider first a simple case of fatigue at two load levels. The life distribution at these two stress levels, S_1 and S_2, are considered known. Figure (B-1 a) shows schematically these distributions. We shall consider these distributions are of the two-parameter Weibull type, with scale parameters (characteristic life) β_1 and β_2. Figure (B-1 b) shows these two distributions in terms of life normalized by its individual characteristic life, β_1 and β_2.

Assume we first load the specimen at load level S_1 to a cycle x_1, and then shift to load S_2 and fatigue to failure. We are interested in finding an equivalent cycle x_2 at the load level S_2 which gives the same amount of fatigue wear and damage as x_1 cycle at load S_1. This is the basic question posed by all phenomenological cumulative damage rule.

There are two alternate approaches in formulating a cumulative damage rule, one is based on percent-life consumed, which is used by Miner's rule and many of the existing rules. The other approaches are based on percent-failure, which is proposed in the present paper.

Miner's Rule - (Percent-Life Damage Rule)

The basic assumption in Miner's rule is that fatigue wear can be characterized by the percentage of life consumed. If we choose to use the Weibull scale parameter (characteristic life) as a reference[1], then Miner's rule is

$$\frac{x_1}{\beta_1} = \frac{x_2}{\beta_2} . \qquad (1)$$

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[1] Miner's rule as originally proposed in 1945, and also as generally applied, does not define clearly the "life" at a given stress. It intended to be the life of the specific specimen involved. Since in actual case, life of a given specimen is never known, people have used some average value of life. We use the characteristic life here for convenience. Similar results may be obtained with mean life.

In other words, if x_1 is equal to 0.5 β_1, we say the specimen has been fatigued to half of its characteristic life. When we next move it to a different stress level S_2, we assume that the specimen has already consumed 0.5 of the characteristic life at S_2. In Figure (1b), this is equivalent of moving from the point $A[x_1/\beta_1, F_1(x_1/\beta_1)]$ vertically to point B on F_2.

Note that, in general, the values of $F_1(x_1)$ are not equal to $F_2(x_2)$. Therefore, using point B to determine x_2 involves a statistical inconsistency. Consider a group of specimens all loaded at S_1 to cycle x_1. At this cycle, let us assume 20% of the specimens failed, or $F_1(x_1) = 0.2$. Now, when we shift to point B on the F_2 curve, the values of $F_2(x_2)$ is, say 0.3. We have 80% of specimen left, while the F_2 curve at point B indicates there should be only 70% left, apparently this is not consistent.

Percent-Failure Damage Rule

In this case, we assume that the degree of fatigue wear is characterized by the percentage of failure of the total population, or the total sample, up to that cycle. If the specimens are loaded under stress S_1 to cycle x_1, we have $F_1(x_1)$ percentage of failure. Now, if we shift the loading to S_2, we assume the same percentage of failure, or,

$$F_2(x_2) = F_1(x_1) \tag{2}$$

This is the basic assumption of the percent-failure rule. In Figure (1b), this is equivalent to moving from point A horizontally to point C. If 20% of the specimens failed under S_1 at x_1, when shifted to S_2, we assume that 20% have failed, regardless of due to which load. This damage rule is based on a group of specimens, or the population, instead of a individual specimen. Ideally, if the failure mechanism of each specimen is known completely, then the behavior of the population can be derived. Since we do not know the failure mechanism, we are forced to study the behavior of the population directly.

If the shape parameter at these two stress levels are the same, $\alpha_1 = \alpha_2$, then the curve, F_1 vs. (x_1/β_1), and the curve F_2 vs. (x_2/β_2) coincide, and the percent-life and percent-failure assumptions are identical. In general, $\alpha_1 \neq \alpha_2$, and they give different results.

For fatigue at more than two load levels, Miner's rule and the present percent-failure rule may be expressed as follows. If x(1), x(2),..., are the cycles applied at stress levels 1,2,... respectively, then Miner's rule states

$$\sum \frac{x(i)}{\beta_i} = 1 \tag{3}$$

where β_i is the characteristic life of the ith loading. The present percent-failure rule will be

$$\sum \Delta F_i(x(i)) = 1 \tag{4}$$

where $\Delta F_i(x(i))$ is the percentage of failure occurred during the ith load. This is equivalent to stating that the sum of the percentage failed at each loading level is one, as shown schematically in Figure B-2. The number of cycles required to fail all specimens is then

$$x = \sum x(i) \tag{5}$$

Life Distribution under Two Loadings

Let the Weibull parameters be α_1, β_1 for load S_1, and α_2, β_2 for load S_2, or

$$F_1(x) = 1 - \exp\left[-\left(\frac{x}{\beta_1}\right)^{\alpha_1}\right] \tag{6}$$

$$F_2(x) = 1 - \exp\left[-\left(\frac{x}{\beta_2}\right)^{\alpha_2}\right] \tag{7}$$

If we load a group of specimens at load S_1 to a cycle x_1 and then switch to load S_2 and fatigue all specimens to failure, the present percent-failure damage rule yields, from Eq. (2),

$$\left(\frac{x_2}{\beta_2}\right)^{\alpha_2} = \left(\frac{x_1}{\beta_1}\right)^{\alpha_1} . \tag{8}$$

This relation implies that for each individual specimen, x_1 cycles at load S_1 is equivalent to x_2 cycles at load S_2.

The life distribution at load S_2, of those specimens that survived x_1 cycles at S_1 (or x_2 cycle at S_2) is

$$F_{2,x_2}(x) = P[X < x | X > x_2] = 1 - \exp\left[-\left(\frac{x}{\beta_2}\right)^{\alpha_2} + \left(\frac{x_2}{\beta_2}\right)^{\alpha_2}\right] \tag{9}$$

Let $y = x - x_2$, where y is the cycle actually endured at stress S_2, Eq. (9) becomes

$$F_{2,x_2}(y) = 1 - \exp\left[-\left(\frac{y+x_2}{\beta_2}\right)^{\alpha_2} + \left(\frac{x_2}{\beta_2}\right)^{\alpha_2}\right] \tag{10}$$

Figure 3 shows the schematic curves of Eqs. (9) and (10).

The complete life distribution of specimens undergoing fatigue at load S_1 to x_1 cycle and then at S_2 to failure can be obtained by shifting the upper portion of the $F_2(x)$ curve to the left. The upper portion of $F_2(x)$ is, as shown in Figure B-3.

$$F_2(x) = 1 - \exp\left[-\left(\frac{x}{\beta_2}\right)^{\alpha_2}\right], \quad x > x_2. \tag{11}$$

Shifting this curve to the left by the amount $(x_2 - x_1)$ gives

$$F_2(x) = 1 - \exp\left[-\left(\frac{x-x_1+x_2}{\beta_2}\right)^{\alpha_2}\right], \quad x > x_1 \tag{12}$$

This equation is also shown in Figure B-3.

The complete life distribution of specimens undergoing fatigue at load S_1 to x_1 cycles and then at load S_2 is, then,

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\beta_1}\right)^{\alpha_1}\right], \quad \text{for } x < x_1$$

$$= 1 - \exp\left[-\left(\frac{x - x_1 + x_2}{\beta_2}\right)^{\alpha_2}\right], \quad \text{for } x > x_1 \tag{13}$$

where

$$x_2 = \beta_2 (x_1/\beta_1)^{\alpha_1/\alpha_2}.$$

Figure B-4 gives a schematic plot of Eq. (13).

Let us restate that if the constant stress life distributions are given by Eqs. (6) and (7), then the life distribution under two stress levels S_1 and S_2 is given by Eq. (13). Once this distribution is known, mean value of cycles applied at each stress level can be calculated.

Comparison with Experimental Data

A series of experiments was conducted to compare with the present analytical results. Tension fatigue tests were used for a unidirectional graphite/epoxy composite. The specimens used and the testing procedure are the same as those described in reference 6. Specimens are cut from composite plates. Specimen 6 and those for the present cumulative damage study are not from the same set of plates. We suspect that there are some differences in the properties between the 12 plates used in reference 6 and the 5 plates used here. Therefore, the values of the fatigue life distribution parameters of the present work are different from those of reference 6. Maximum likelihood estimation of these data gives the Weibull parameters as follows:

at stress level S_ℓ = 150 ksi,

$\alpha_\ell = 0.36$, $\beta_\ell = 3.9 \times 10^7$ cycles, $\mu_\ell = 17.8 \times 10^7$ cycles

at stress level S_h = 180 ksi

$\alpha_h = 0.37$, $\beta_h = 42 \times 10^3$ cycles, $\mu_h = 176 \times 10^3$ cycles

We have conducted two types of cumulative damage fatigue experiments. The first is a "low-high" test; the specimens are first subjected to fatigue at the lower stress S_ℓ to 458900 cycles (x_1), those survived are then fatigued at the higher stress S_h. The other experiment is the "high-low" test, the specimens are subjected to S_h for 3570 cycles, and then fatigued at S_ℓ to failure.

The life distributions as calculated from Eq. (13) for both cases are shown in Figs. B-5, B-6. The experimental data are also plotted on these figures for comparison. The median rank formula is used in plotting the cumulative distribution F position of the data points [7]. Even though the number of specimens is rather small, the data points are in general agreement with the theory. This series of tests has not been completed yet; results with a large number of specimens will be presented shortly.

Comparison with Another Set of Experiments

Awatani, Skiraishi and Tsukahara [8] recently conducted some fatigue experiments and applied Miner's rule to their data. Their cumulative damage experiments are very similar to ours presented above, a Low-high test, and a High-low test. They found that, in general, the sum $(n_1/N_1) + (n_2/N_2)$ for the Low-high test is larger than unity, which is predicted by Miner's rule. In other words, the Low high tests give higher life than that given by Miner's rule. For the High-low tests, the general results show lower life than that predicted by Miner's rule. The authors offered some explanation for these discrepancies by crack initiation, strain-aging and other fracture mechanics reasons. It will be shown here that the experimental results of reference 8 can be made to agree with the present theory if the life distributions at the two stress levels have different shape parameter. This is a phenomenological approach, no fracture mechanics assumption is needed.

In reference 8 the test results are presented in the form of $(1 - n_2/N_2)$ vs (n_1/N_1) curves and points. The term $(1 - n_2/N_2)$, which is called the damage ratio, should be equal to n_1/N_1 if Miner's rule is correct.

Note that reference 8 treated all quantities as deterministic, and used the average from three tests to represent one data point. In order to apply our theory, we have to redefine all terms such as n_1 and N_1.

The number x_1, or n_1, is the maximum number of cycles applied a stress level S_1 to all specimens. But, not all specimens in a population can survive to x_1 cycle. Therefore, the number of cycles at stress level S_1 is a random variable, and its expected value, or mean, should be used. Using the notation $E(x_1)$ for the expected value of x_1, we have,

$$E(x_1) = \int_0^{x_1} x\, f_{1,x_1}(x)dx \qquad (14)$$

where

$$f_{1,x_1} = \frac{d\, F_{1,x_1}(x)}{dx} \qquad (15)$$

and

$$F_{1,x_1}(x) = P(X < x | X < x_1) = \frac{1 - \exp\left[-\left(\frac{x}{\beta_1}\right)^{\alpha_1}\right]}{1 - \exp\left[-\left(\frac{x_1}{\beta_1}\right)^{\alpha_1}\right]} \qquad (16)$$

After carrying out the integration, Eq. (14) becomes

$$E(x_1) = \frac{\beta_1 \Gamma\left(\frac{1}{\alpha_1} + 1\right) \gamma^* \left(t_1, \frac{1}{\alpha_1} + 1\right)}{t_1^{-\left(\frac{1}{\alpha_1} + 1\right)} [1 - \exp(-t_1)]} \tag{17}$$

where Γ is the gamma function, γ^* the incomplete gamma function (see for instance, [9]) and

$$t_1 = \left(\frac{x_1}{\beta_1}\right)^{\alpha_1}$$

The number of cycles under the stress S_2, $y = x(2) = x - x_2$, is also a random variable, with expected value $E(y)$, or,

$$E(y) = \int_0^\infty y\, f_{2,x_2}(y)\, d(y) \tag{18}$$

where

$$f_{2,x_2}(y) = \frac{d\, F_{2,x_2}(y)}{d(y)} \tag{19}$$

Using Eq. (10), and carrying out the integration, we obtain

$$E(y) = \beta_2 \exp(t_2) \left[\Gamma\left(\frac{1}{\alpha_2} + 1\right) - \frac{\Gamma\left(\frac{1}{\alpha_2} + 1\right) \gamma^* \left(t_2, \frac{1}{\alpha_2} + 1\right)}{t_2^{-\left(\frac{1}{\alpha_2} + 1\right)}}\right] - x_2 \tag{20}$$

where

$$t_2 = \left(\frac{x_2}{\beta_2}\right)^{\alpha_2} .$$

Let μ_1 and μ_2 be the mean life at S_1 and S_2, respectively. Then, our present interpretation of Miner's rule should be

$$\frac{E(x_1)}{\mu_1} + \frac{E(y)}{\mu_2} = 1 \tag{21}$$

or

$$1 - \frac{E(y)}{\mu_2} = \frac{E(x_1)}{\mu_1} \tag{22}$$

The expression $1 - E(y)/\mu_2$ should replace the term $1 - n_2/N_2$ used in reference 8, and $E(x_1)\mu_1$ replaces n_1/N_1 in reference 8. In Fig. B-7, the experimental points are $(1 - \frac{n_2}{N_2})$ vs $(\frac{n_1}{N_1})$, as obtained in reference 8. The miner rule straight line is obtained by directly equating $[1 - E(y)/\mu_2]$ to $E(x_1)/\mu_1$. The curves of the present theory are plotted by calculating $E(x_1)$ from Eq. (17), and $E(y)$ from Eq. (20).

The mean life at the low stress level (127 N/mm^2) is given in [8] as 2.2×10^6 cycles; the mean life at the high stress level (169 Nmm^2) is 7×10^4 cycles. We have assumed that Weibull shape parameters at these two stress levels are 5.0 and 2.5, respectively, the corresponding values of $E(x_1)$ and $E(y)$ are calculated from Eqs. (17) and (20). It can be seen that the curves of the present theory agree very well with the test data points. For the Low-high tests, the test points are below the Miner's rule line, indicating longer life; the High-low tests are above the Miner's rule line.

The original test values of every experiment are not given in reference 8, only averages are given. Therefore, the shape parameters cannot be estimated. The values of 5.0 and 2.5 are selected just to demonstrate that the deviation of the experimental results from Miner's rule could be due to statistical distribution.

CONCLUDING REMARKS

1. Miner's rule as expressed in conventional forms, such as Eqs. (1) or (3), has statistical ambiguity, and cannot be compared with test data.
2. A statistically meaningful form of Miner's rule may be expressed as Eq. (21). It is shown that the left hand side of Eq. (21) could be larger, or smaller than unity. Therefore, Miner's rule is not always correct.
3. Fatigue life distribution under two different stress levels consecutively can be expressed as in Eq. (13).
4. The method used here can be generalized to cases of more than two stress levels.
5. The present percent-failure damage rule shows good agreement with two sets of experimental data. Additional test data are needed to evaluate its merits.

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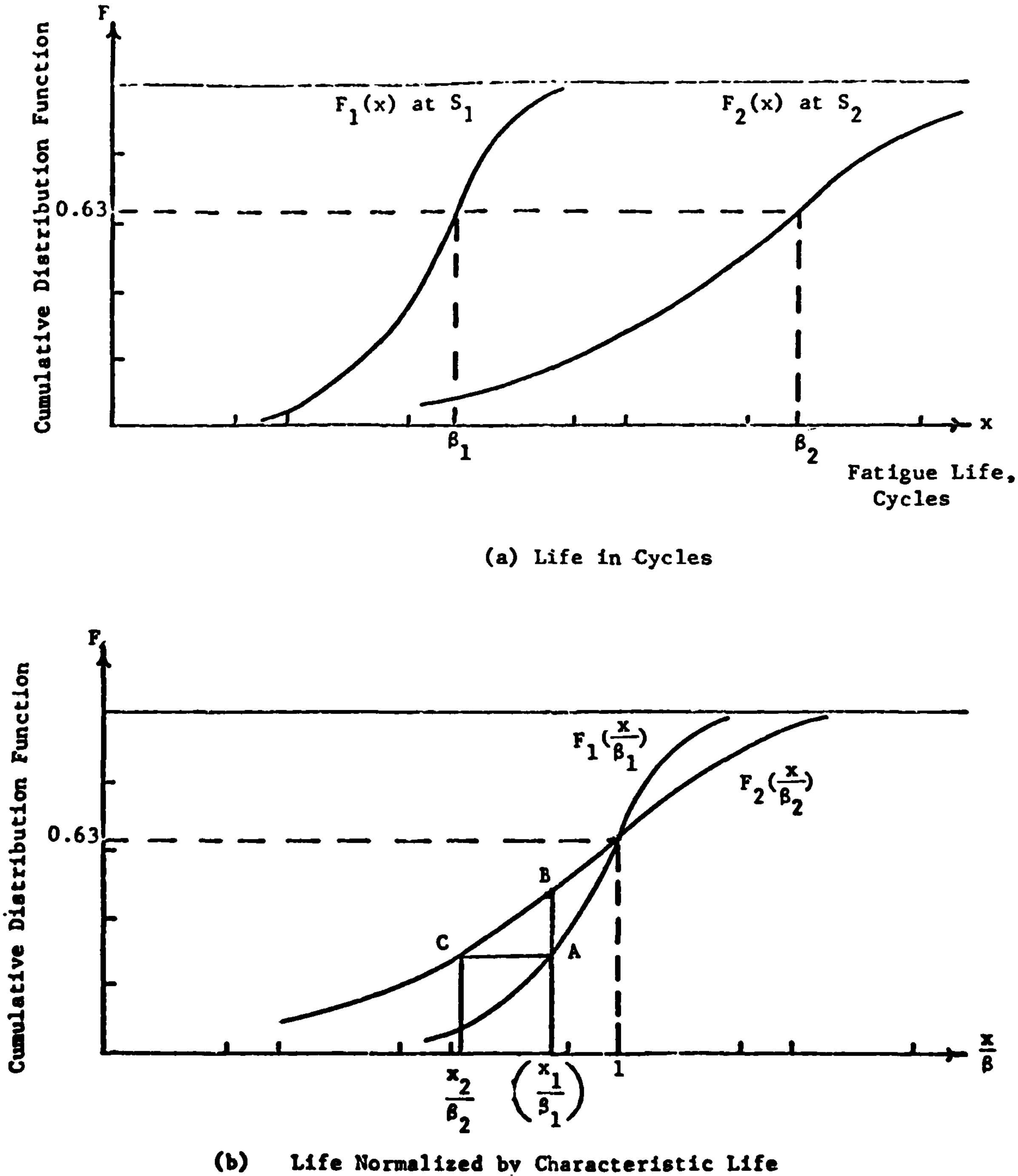

Figure B-1. Cumulative Distribution Functions of Fatigue Life at Two Load Levels

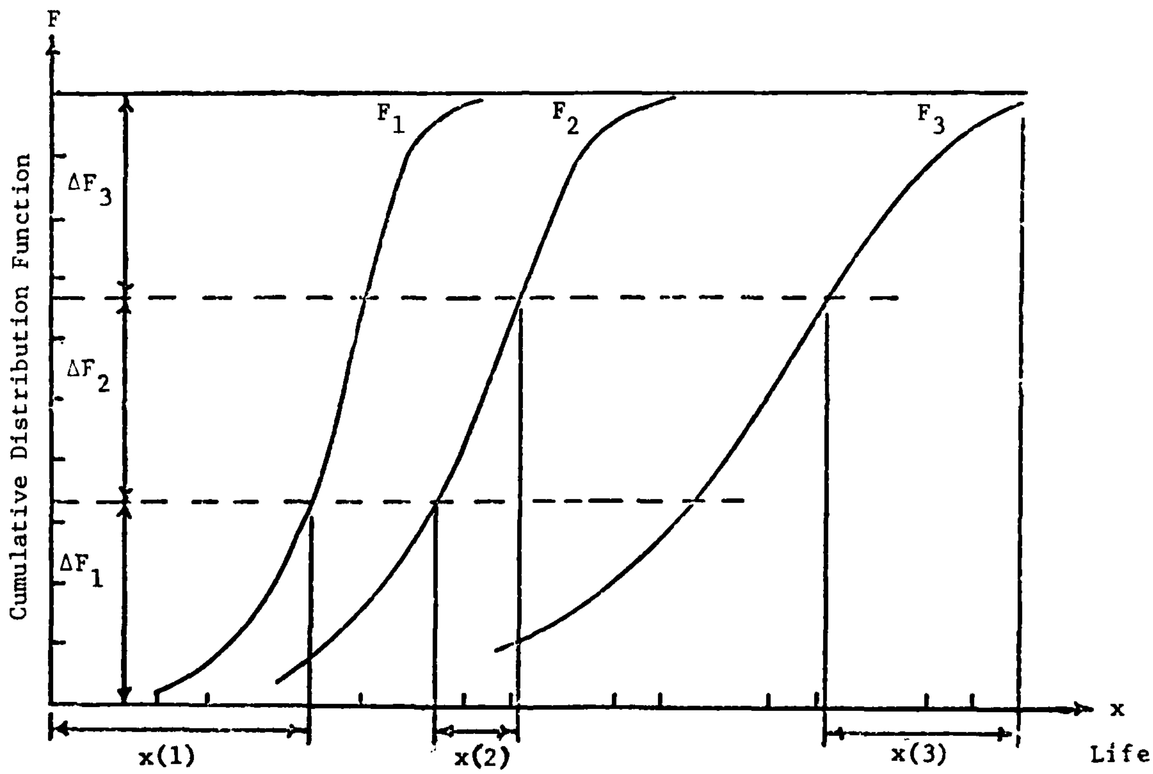

Figure B-2 Distribution Curves Showing Cumulative Damage by the Present Percent-Failure Rule (Eq. 4 and 5).

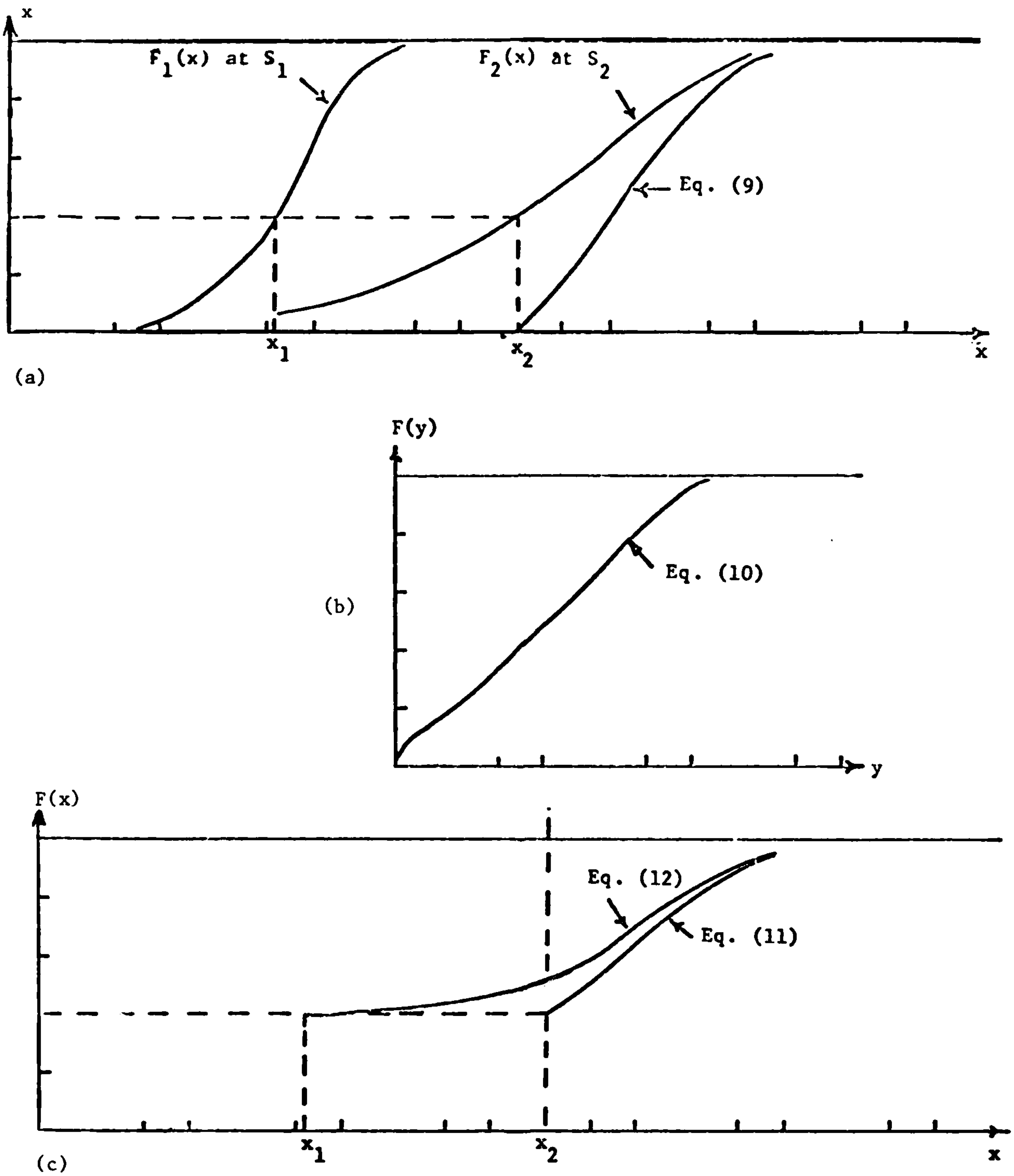

Figure B-3 Cumulative Distributions of Fatigue Life:

(a) At Stress Levels S_1 and S_2, and at S_2 for Specimens Survived x_2, Eq. (9).

(b) At S_2 in Terms of Additional Life y, and

(c) At S_2 in Terms of x.

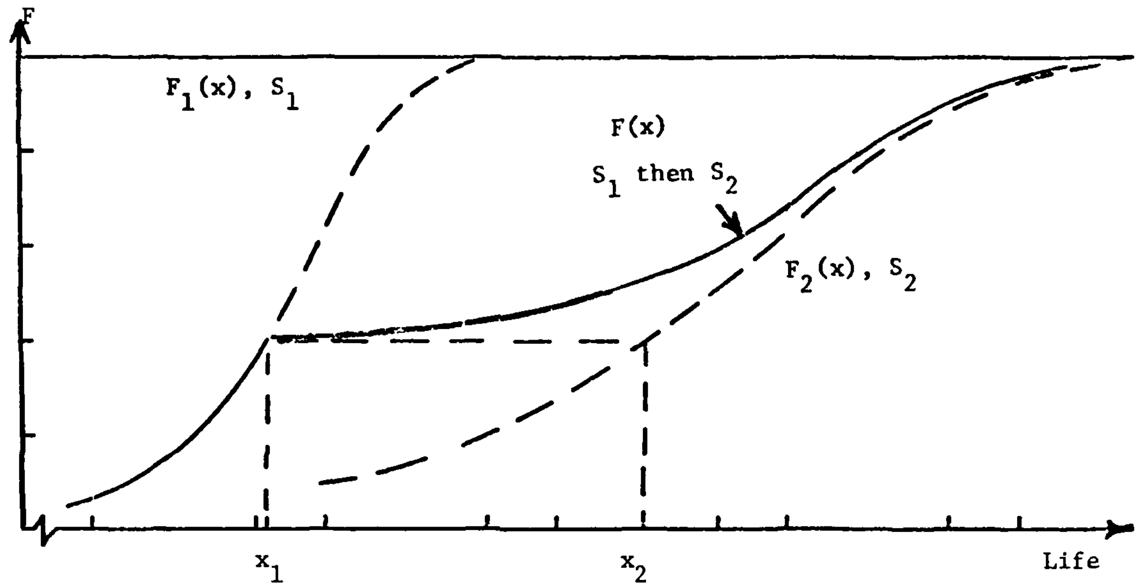

Figure B-4 Life Distribution of Fatigue at Stress Level S_1 and then S_2

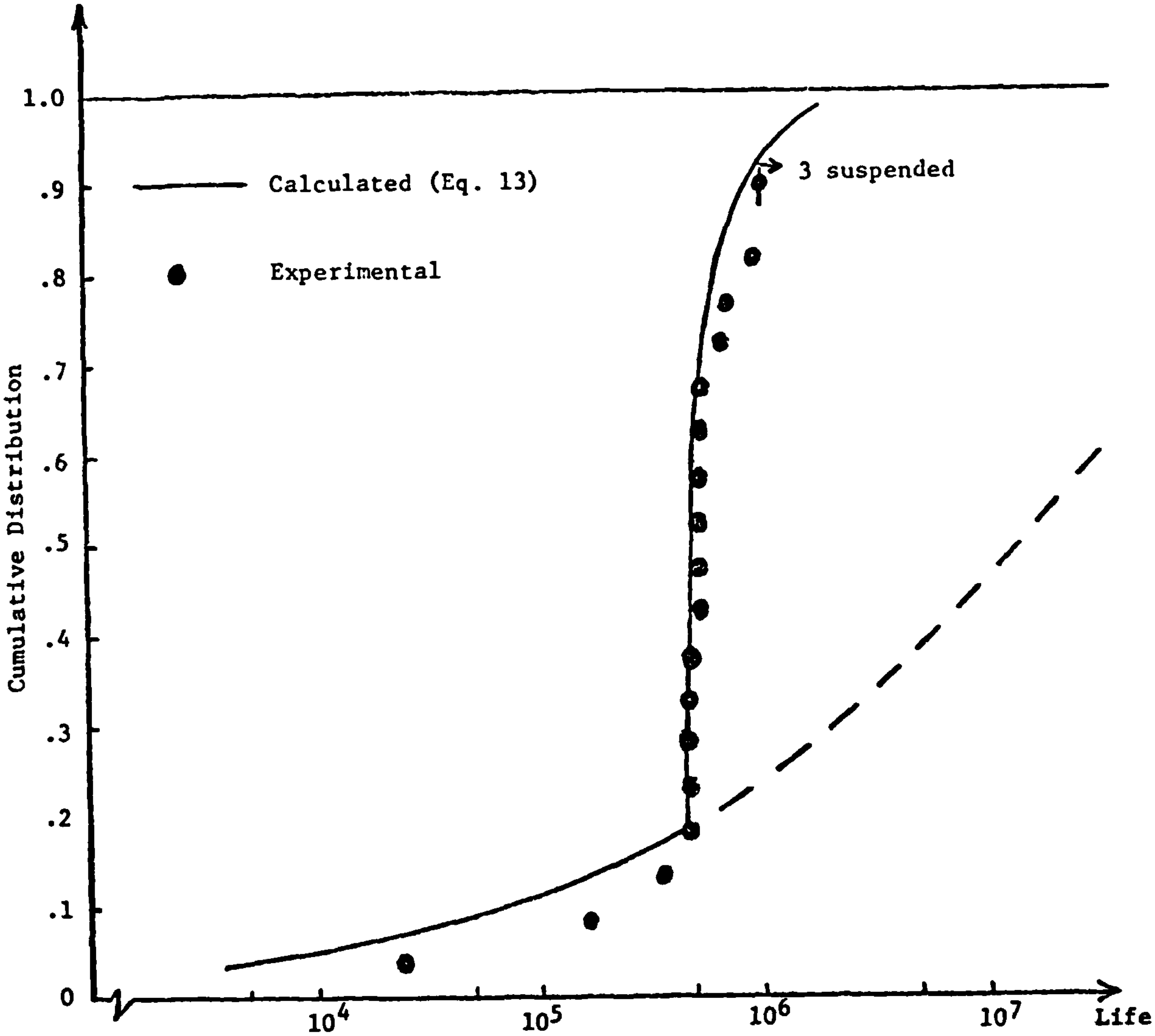

Figure B-5 Comparison between Calculated and Experimental Cumulative Distribution of Fatigue Life of Specimens Subjected to Low-High Loading (Low stress to 4.59×10^5 cycles, then high stress to failure)

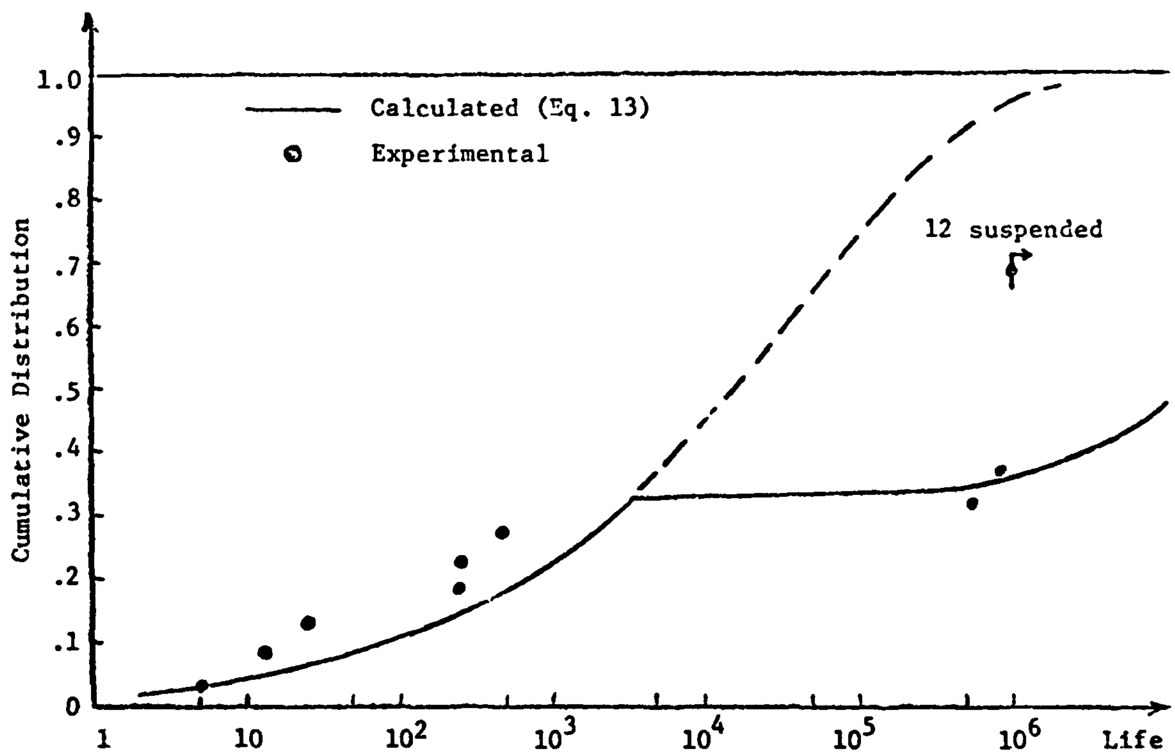

Figure B-6. Comparison between Calculated and Experimental Cumulative Distribution of Fatigue Life of Specimens Subjected to High-Low Loading (High stress to 3.6×10^3 cycles then low stress to failure)

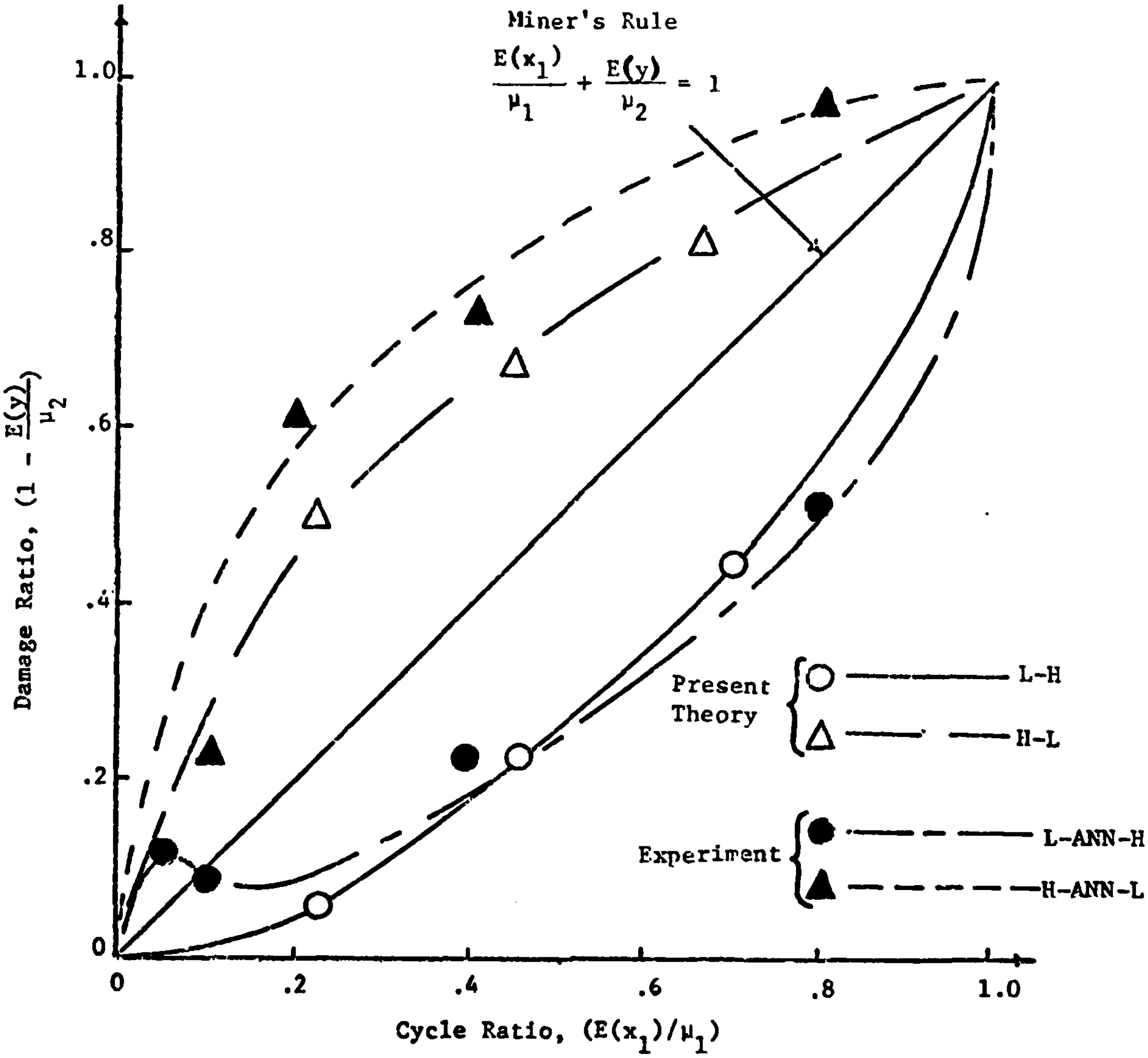

Figure B-7. Comparison of the present Percent-Failure Rule (Eq. 22) with Experimental Data (Ref. [8]).

*U.S.Government Printing Office: 1980 — 657-084/11

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